ABSTRACTS OF PAPERS PRESENTED AT THE 46th CONFERENCE OF THE INDIAN MATHEMATICAL SOCIETY

(Bangalore : December 27-29, 1980)

ABSTRACTS OF PAPERS PRESENTED AT THE 46th CONFERENCE OF THE INDIAN MATHEMATICAL SOCIETY

(*Bangalore : December* 27-29, 1980)

1. ALGEBRA, LATTICE THEORY

1. **Periodic modules for the group SL$(2,p^n)$:** *A. Vincent Jeyakumar, Madurai.*

Let G be a finite group and let K be a field of characteristic $p>0$. A finite dimensional KG-module M is said to be periodic if there exists an exact sequence

$$0\to M\to P_{n-1}\to\ldots\to P_0\to M\to 0$$

where each P_i is a projective KG-module.

It is known that if the Sylow p—subgroup of G is cyclic and K is the algebraic closure of its prime subfield, then any KG-module is periodic.

We prove in this paper the following result in the non-cyclic case, which is a generalisation of a result due to Alperin.

"Let G be the group SL$(2, p^n)$ (p and odd prime) and Let K be the algebraic closure of its prime subfield. Then any irreducible KG-module whose dimension is divisible by p^{n-1} is periodic."

2, **On N-projective covers and N-injective envelopes :** *Anil K. Gupta, Delhi.*

In this note we have defined N-projective covers and N-injective envelopes of modules. In case a given module M has a projective cover, the existence and essential uniqueness of an N-projective cover is proved. Existence of N-injective envelopes is also established.

3. **Generalized multiplication modules :** *R.K. Jain.*

All rings considered here are commutative and have a unity. All modules are unital left modules. A submodule N of an R-module M is said to be a multiplication submodule if whenever a submodule K is contained in N, there is an ideal A of R such that $K=AN$. M is said to be a multiplication module if every submodule of M is a multiplication submodule. In

case every proper submodule of M is a multiplication submodule, M is said to be a generalized multiplication module. A module M is said to be a pseudocancellation module ((PC)-module) if for every proper ideal A of R, $AM<M$.

Theorem. Let M be a (PC)-module over a quasi-local ring R. Then M is a generalized multiplication module if and only if M satisfies one of the following :

(a) M is a uniserial module.

(b) M has a unique infinite descending chain of submodules without proper refinements.

(c) M possesses maximal submodules and through each maximal submodule, there passes a unique composition series of M.

(d) M possesses maximal submodules and all the non-zero submodules contained in a maximal submodule form an infinite descending chain without proper refinements.

4. **Minimal element in certain partially ordered semi-rings :** *K. Venu Raju, Tirupathi.*

In this paper the author studies some properties of minimal element and its influenced properties in certain partially ordered semi-rings. The main theorem of the paper :

Set $(S,+,.,\leqslant)$ be a totally ordered semi-ring with a minimal element and satisfying $S=S+S$, $(S, +)$ is *p.t.o.* and $(S, .)$ is *r.n.t.o.* . Then $(S, +)$ is a chain under dual ordering and the minimal element is additive identity.

The author notices that, in the above theorem the ordering is unique and each of the conditions is essential,

5. **On generalized multiplication modules :** *Surjeet Singh and Qazi Zameeruddin.*

As defined by Mehdi, a module M_R is said to be a multiplication module if for every pair of submodules K and N of M, $K\subseteq N$ implies $K=N A$ for some ideal A of R.

Singh and Mehdi introduced the concept of a generalized multiplication module (GMM). A module is called GMM if for every pair of proper submodules K and N of M, $K\subseteq N$ implies $K=NA$ for some ideal A of R.

Following theorems are proved :

Theorem 1. Let K be a finitely generated faithful GMM over a quasi-local ring (R, M). Then one of the following holds :

(I) K is a multiplication module and R is a multiplication ring.

(II) K/KM is a direct sum of two simple modules. KM is a multiplication module. For any $x \in K$, $x \notin KM$, $xM = KM$.

Theorem 2. Let K be a faithful module over a Noetherian ring R. Then K is a GMM over R if and only if one of the following holds :

(i) K is a GMM with R being an integral domain.

(ii) K is a multiplication module.

(iii) R is a local ring with maximal ideal M such that K/KM is a direct sum of two simple modules, KM is a multiplication module and for every $x \in K$, with $x \notin KM$, $xM = KM$.

6. **On local adjointness** : *Arun K. Srivastava, Varanasi.*

Several constructions in mathematics *e.g.*, the algebraic closure of a field and the universal covering spaces, have a kind of universal property not similar to the usual universality. This leads to the concept of local adjointness of functors [KAPUT, locally adjunctable functors, Ill. J. Math. 16 (1972), 86-94] generalizing the concept of adjoint functors. In the passage from adjointness to local adjointness, several pleasing properties (of adjointness) are lost but some remain intact, sometimes, with slight modifications. Some such properties are pointed out. A particularly interesting observation is : Every locally right adjunctable functor $F : A \to C$ induces a monad in A.

7. **A Note on Strong Krull Rings** : *Saroj Malik.*

Motivated by the problem of finding properties common to the Prüfer domains and the Noetherian domains which make them stably strong S-rings, we introduce a new class of rings, the strong Krull rings (SK-rings). A ring R is a strong Krull ring if and only if R/P is a Krull domain for each prime ideal P of R. We prove :

Theorem 1. If R is a Dedekind domain and $R[X]/Q$ is integrally closed for all uppers Q of (0), then $R[X]$ is a strong Krull ring.

Theorem 2. If R is an SK-ring and $a \in R$, a nonunit nonzero divisor, then, if P is a prime ideal of R minimal over aR, $ht\ P \leqslant 1$.

Theorem 3. If R is an SK-ring and $A=(a_1, a_2, \ldots, a_n)$ a finitely generated ideal of R, then for any minimal prime P of A, $ht\ P \leqslant n$.

Theorem 4. Any integral extension and the integral closure of an SK-ring in its total quotient ring is a strong S-ring.

Theorem 5. An SK-ring is a GB-ring.

8. **Prufer domains as GB-rings and altitude-formula** : *Saroj Malik, Delhi.*

Here in this paper we are concerned with chains of prime ideals in a Prüfer domain and in polynomial rings over a Prüfer domain. We have shown that a Prüfer domain R is a GB-rlng and so is $R[X]$, the polynomial ring over it in the indeterminate X. We also prove that if prime ideals in a Prüfer domain hsve finite height, then the altitude formula holds.

9. **p-semi-ideals and restricted ideals in Rickart*-semigroups** : *N.K. Thakare and S.K. Nimbhorkar.*

In this note one more characterization of Baer*–semigroups is given. The concepts of p-ideals and restricted ideals in Rickart* -rings are generalized to p-semi-ideals and restricted ideals in Rickart* -semigroups. Some properties of p-semi-ideals in Rickart* -semigroups are established. A characterization of restricted ideals in Rickart* -Semigroups is accomplished. As a final result we establish a one-to-one correspondence between the set of p-semi-ideals and the set of restricted ideals of a Rickart* -semigroup satisfying the condition, that the left projection of every element is equivalent to its right projection.

10. **On an m-semigroup** : *L. Lakshmanan, Ananth K. Atre and T. Ramesan.*

An M-semigroup M is a semigroup which satisfied the following conditions : (i) there is atleast one $e \in M$, such that $ex=x$, for all $x \in M$; and (ii) for each $x \in M$, there is a unique left identity e such that $xe=x$. A semigroup S is said to be a semi-inflation of a subsemigroup B if there is a homomorphism f from S onto B that fixes the elements of B. In that case, we say B is a semideflation of S. A semigroup S is said to be a fine band A of semigroups (i) f^{-1}, $i \in A$ if there is a homomorphism $f: S \rightarrow A$, from S onto a band A such that $i\,j=k$, $(i, j, k \in A)$; (i) f^{-1}. (j) $f^{-1} \subseteq [(k) f^{-1}]^2$

We observe that an M-Semigroup is isomorphic to a direct product $R\times S$, where R is a right singular semigroup and S is a semigroup with a two sided identity and further we prove that an M-Semigroup M is a fine band R of isomorphic left ideals of M, which are semideflations of M.

11. Base of factorization in a semigroup : *Ananth K. Atre, A. Jayalakshmi.*

Let S be a semigroup and a an element of S^2. By the base of factorization of a, we mean the subset $\bar{a}=\{(x,y) \mid x\in S,\ y\in S,\ xy=a\}$ of $S\times S$. By a regular decomposition of a semigroup S we mean a decomposition $\bigcup_{i\in A} S_i$ of S such that $\{S_i \mid i \in A\}$ defines a congruence relation on S.

In this paper, we obtain the following theorems.

Theorem 1. Let S be a semigroup. Then each base of S will be a subsemigroup of $S\times S$ if and only if S is an inflation of a regular band.

Theorem 2. Let S be a semigroup. Then $S\times S$ is regularly decomposable into the bases of S if and only if S is an inflation of a normal band.

12. On nearly right simple semigroups : *A. Jayalakshmi, Ananth K. Atre, T. Ramesan.*

R. Scozzafava has defined a nearly right simple semigroup S as a semigroup that satisfies the conditions

(i) $S^2=S$
(ii) for any a,b,c in S, there exists s in S such that $abs=ac$.

In this paper we obtain the following results.

Theorem 1. In a semigroup S, the following are equivalent.

1.1 S is a nearly right simple semigroup.

1.2 aS is a minimal right ideal of S for all a in S.

1.3 aS is right simple for all a in S.

1.4 S is a left singular semigroup of right simple semigroups.

Theorem 2. In a nearly right simple semigroup S, the following are equivalent.

2.1 S contains an idempotent.

2.2 aS is a right group for all a in S.

2.3 S is a left singular semigroup of right groups.

2.4 S is a union of groups.

13. **Natural pre-congruences :** *K.S. Harinath, Banglore.*

The natural occurrence of a reflexive transitive relation which is simultaneously left and right compatible (=pre-congruence) on a right (or left) inverse semigroup is proved. This pre-congruence reduces to a skew-congruence (*i.e.* to the natural partial order) whenever the semigroup considered is inverse, and is the identical-congruence over a group. Various results pertaining to this pre-congruence are incorporated and a plausible generalization to regular semigroups is attempted. This is defined by : *a $ b in S iff $aa'=ab'$* with usual notations.

14. **Certain results on regular semigroups :** *K.S. Harinath and K.N. Kamalamma, Banglore.*

This paper is concerned with a few subclasses of the class of regular semigroups which require further attention and contains a collection of results. For instance, the maximum idepotent-separating congruence on a completely regular semigroup is analysed in terms of the unique commuting inverses, which generalizes the corresponding results of Clifford semigroups and of completely regular orthodox semigroups. On a general semigroup S, an equivalence relation K is defined by setting $a\ K\ b$ in S iff $aS^1a=bS^1b$, which is shown to contain the Green's relation H on S and which coincides with H whenever the semigroup S is regular. It is noted that on a Clifford semigroup, K is a congruence. Also, on an arbitrary semigroup, a K-class can contain at most one idempotent.

15. **On finitely generated n-commutative semigroups :** *V.R. Chandran and Vimala Lakshmanan, Madras.*

A semigroup S is said to be n-commutative if the following identity $x_1x_2....x_n=x_2x_3....x_nx_1$ holds in S. In this paper we generalize the results of L. Redei to the n-commutative semigroups.

Theorem. All finitely generated n-commutative semigroups are finitely presented.

Various lemmas are proved in sequel to prove the above theorem.

16. **Characterization of the Boolean algebra of switching functions by (0, 1) matrices and application to minimization :** *Ponnammal Natarajan Perarignar Anna University of Technology.*

Every switching function of n variables is a mapping from $Z_2^n \to Z$ and

these switching functions form a 2^{2^n} element Boolean algebra when addition multiplication (not composition) and complementation of these functions are defined suitably. In this paper an unique $(o, 1)$ matrix of order $2^r \times 2^s$ $(r+s=n)$ is associated with every switching function of n variables. This representation is shown to be an isomorphism of the Boolean algebra of switching functions of a variables into the set of $2^r \times 2^s$ $(o, 1)$ matrices considered. This matrix representation leads to the development of a new algorithm for minimization of switching functions. The advantages of this new algorithm are as follows :

1. The number of variables, n, does not affect the procedure (a definite advantage over Karnaugh map method).

2. We can use the form in which the function is expressed without obtaining the equivalent canonical form (a positive advantage over McClusky method).

17. Essential and normal extensions of regular semigroups : *K. S. S. Nambooripad and S. Radhakrishnan Chettiar, University of Kerala, Kariavattom.*

In this paper we show that the concepts of normal and essential extensions of inverse semigroups introduced by *M*. Petrich ['Extensions normales de demi-groupes inverses' (submitted for publication) and 'The Conjugate hull of an inverse semigroup', (submitted for publication)] can be generalised to regular semigroups. We obtain an explicit construction of the normal hull of a regular semigroup S and show that S has a maximum normal extension if and only if the metacenter of S is idempotent. We also describe some applications of normal hulls of regular semigroups in describing **H** -co-extensions of regular semigroups.

18. A structure theorem for E-unitary pseudo-inverse Semigroups : *R. Veeramony, University of Kerala, Kariavattom*

McAlister [Groups, semilattices and inverse semigroups, I and II, Trans. Amer. Math. Soc., 192 (1974), 227-244 and 196 (1974), 351-370] has shown that every E-unitary inverse semigroup (equivalently, an inverse semigroup with injective structure mappings) can be constructed from a partially ordered set **X** containing a semilattice Y as an ideal, and a group G acting on **X**. Pastijn [Structure of pseudo-inverse semigroups, submitted for publication] has generalized this result and has shown that every E-unitary pseudo-inverse semigroup (i.e., a pseudo-inverse semigroup with

injective structure mappings) is a subsemigroup of a rectangular band of E-unitary inverse semigroups. In this paper we obtain a simpler construction of E-unitary pseudo-inverse semigroups similar to McAlister's construction of E-unitary inverse semigroups. We prove that every E-unitary pseudo-inverse semigroup can be constructed from a partially ordered set **X** containing a given family $\{Y_i\}$ of semilattices as ideals and a completely simple inductive groupoid G acting on **X** on both sides.

19. A structure theorem for combinatorial regular semigroups : *A.R. Rajan, University of Kerala, Kariavattom.*

In [Structure of combinatorial regular semigroups by *K.S.S.* Nambooripad and A.R. Rajan, Quart. J. Math. Oxford 29 (1978) 489-504] the structure of combinatorial regular semigroups is described in terms of a biordered set E and a functor from the preorder (E, ω) to the category of combinatorial Rees groupoids. This construction depends on the biordered set E whose structure is assumed to be known. In this paper we give a structure theorem for combinatorial regular semigroups in terms of a strictly skeletal small category D, two set valued functors P_1 and P_2 and a subdirect product $\triangle$ of P_1 and P_2. This theorem in particular yields a structure theorem for combinatorial biordered sets and a structure theorem for bands which is different from the one given by Petrich ["Topics in Semigroups", Pennsylvania State University, 1966]. We also describe homomorphisms of combinatorial regular semigroups in terms of transformations of the corresponding functors.

20. Independence of axioms for biordered sets : *S. Premchand, Department of Mathematics, Regional Engineering College, Calicut.*

The set of idempotents of a regular semigroup was characterized by Nambooripad [Structure of Regular Semigroup I, Memoirs Amer. Math. Soc., No. 224 (1979).] abstractly as a regular biordered set. In this paper we first present a simplified version of Nambooripad's axioms for a biordered set. Even in this simplified set of axioms, there are in effect seven axioms. As a further simplification we give an alternate set of axioms which consists of five self-dual axioms. We also show that the two sets of axioms given in this paper are independent.

21. Idempotent generated free semigroups : *S. Premchand, Department of Mathematics, Regional Engineering College, Calicut,*

F. Pastijn has given two constructions of free idempotent generated semigroups, $S^*(E)$ and $\tilde{S}(E)$ where E is the set of idempotents of a semi-

group. (See F. Pastijn, The biorder on the partial groupoid of idempotents of a semigroup). In this paper we generalize these constructions to the case in which E is an abstract biordered set defined by Nambooripad [Structure of Regular Semigroups I, Memoirs Amer. Math. Soc., No. 224 (1979).] We show further that these constructions are functorial, and the functors S^* and $\tilde{S}$ arising in this way are left adjoints of suitable forgetful functors. We also show that $\tilde{S}$ restricted to the category of regular biordered set is shown to be naturally equivalent to the functor I_0 of *K.S.S.* Nambooripad. We also obtain an explicit form of this natural isomorphism.

22. **Regular partial bands :** *S. Premchand, Department of Mathematics, Regional Engineering College, Calicut.*

In this paper we give the construction of all regular partial bands determined by a biordered set E in terms of closed and effective set of E-squares in E and the free idempotent generated regular semigroups. This construction is also functorial and the functor U obtained in this way is the ieft adjoint of the forgetful functor B that sends a regular semigroup S to its regular partial band.

23. **On n-commutative emigroups :** *V.R. Chandran and Vimala Lakshmanan Ramanujan Institute, Madras-5.*

A semigroup $<S, *>$ is said to be n-commutative if a fined integer $(n \geqslant 2)$ such that for any n-elements in S, the following identity $X_1X_2 \ldots X_n = X_2X_3 \ldots X_nX_1$ holds. A semigroup $<S, *>$ is said to be nowhere n-commutative if the equality $x_1x_2 \ldots x_n = x_ix_{i+1} \ldots x_nx_1x_2 \ldots x_i$ $(i=2. 3, \ldots n)$ where $x_i (i=1, 2, \ldots n) \varepsilon S$ implies $x_1 = x_2 = \ldots = x_n$. The following theorems are proved in this papers.

Theorem 1. An n–commutative semigroup which is also $(n-1)$-quasi commutative is commutative.

Theorem 2. A semigroup $(S, *)$ is nowhere n-commutative if and only if it is a band without zero in which every element is primitive or $|S| = 1$.

Theorem 3. A semigroup is nowhere n-commutative iff any two elements are inverses of each other.

Theorem 4. A rectangular band is nowhere n-commutative.

Theorem 5. Any cancellative n-commutative semigroup can be embedded in a group.

24. Submaximal ideals in a Boolean like ring : *V. Swaminathan, Waltair.*

A Boolean like ring is a commutative ring with unity in which $2x=0$ and $xy(1-x)(1-y)=0$ hold for all elements x,y of the ring. A submaximal ideal is one which is covered by a maximal ideal of the ring. In this paper, we attempt to characterise submaximal ideals of a Boolean like ring R and prove that an ideal I of a Boolean like ring R is submaximal if and only if R/I is either a four element Boolean ring or the four element Boolean like ring. We also prove that if $\bar{X}$ denotes the set of all primary submaximal ideals of R and for each nilpotent $n \in R$, $\bar{X}_n=\{P \in \bar{X} \mid n \notin P\}$, then $(N,+)$ is isomorphic to the Boolean group $(\{\bar{X}_n\}_{n \in N}, \triangle)$ where N is the nilradical of R. Further, we establish that every maximal subring of R is a dual subring of R generated by a submaximal ideal of R and conversely, and that, every dual subring of R is a join of points and meet of hyporplanes in the lattiee of all subrings of R.

25. A structure theorem for pseudo-semilattices : *A.R. Rajan, University of Kerala, Kariavattom.*

A biordered set E is called a pseudo-semilattice if the sandwich set of any pair of elements in E contains exactly one element. Meakin and Pastijn [Structure of pseudo-semilattices, submitted for publication] have given a structure theorem for pseudo-semilattices in terms of semilattices and homomorphisms of semilatttices. In this paper we obtain an alternate structure theorem for psedo-semilattices. Let I and $\wedge$ be two partially ordered sets. A subset $\triangle$ of $I \times \wedge$ is said to be a locally isomorphic subdirect product of I and $\wedge$ if the projections P_I and $P_\wedge$ are surjective and their restrictions to principal ideals of $\triangle$ are order-isomorphisms. In this paper we show that there exists a one-to-one correspondence between the reflective subcategories of the preorder $I \times \wedge$ that are locally isomorphic subdirect products of I and $\wedge$ and psedo-semilattices E such that $E/R \simeq I$ and $E/L \simeq \wedge$. We also deduce a structure theorem for combinatorial pseudo-semilattices and in particular a structure theorem for normal bands as corollaries.

26. Lindelof generated and Lindelof atomistic lattices : *N.K. Thakare and M.P. Wasadikar.*

In this note concepts of Lindelof generated lattices and Lindelof atomistic lattices are introduced and their characterizations have been obtained.

27. Congruence extension property of certain classes of algebras with pseudocomplementation : *(Written jointly with V. Ramam).*

In his paper entitled "Principal congruences of pseudocomplemented semilattices and congruence extension property" Proc. Amer. Math. Soc. Vol. 73 (1979), 308-312, H.P. Sankappanavar has shown that the class of pseudocomplemented semilattices enjoys the congruence extension property using the fact that the class of semilattices enjoys the congruence extension property. He made a remark at the end of his paper that it would be interesting to see if the congruence extension property of pseudocomplemented distributive lattices can be deduced from that of pseudocomplemented semilattices and in the present paper we actually do the same. Our proof is not only fairly simple but also its slight and natural modification in the case of pseudocomplemented distributive lattices yields the well known result that the class of pseudocomplemented distributive lattices enjoys the congruence extension property and simplified a lot of computation and involvedness in the proof of G. Gratzer in his book Lattice Theory, First concepts and distributive lattices, W.H. Freeman and Co., 1971. Further its natural modification in the case or distributive double p-algebras yields not only the result that this class enjoys the congruence extension property but also simplified the proofs given by the earlier authors T. Katrinak and R. Beazer.

28. Equivalence of M-symmetry and semi-modularity in lattices : *Parameshwara Bhatta. S.*

G Gratzer has posed the following problem :

For Algebraic lattices, is M-symmetry equivalent to semimodularity ? A negative solution to this problem is given with the help of a counter-example. Moreover, the equivalence of semimodularity and M-symmetry in a strongly atomic upper continuous lattice is established. From this it is inferred that for algebraic strongly atomic lattices M-symmetry is equivalent to semimodularity.

29. Supermodular lattices : *Iqbalunnisa and W.B. Vasantha.*

Distributive lattices and modular lattices are the two well known equational classes of lattices. Another equational class of lattices called supermodular lattices of order 3 has been introduced by Iqbalunnisa in [3]. It is shown in [3] that this equational class lies between the equational class of modular lattices and the equational class of distributive lattices. In this paper, we choose to term this class of lattices as supermodular lattices and investigate their nature.

A lattice is termed supermodular if it satisfies the identity $(a+b)(a+c)(a+d)=a+bc\ (a+d)+db(a+c)+cd(a+b)$ for all a,b,c,d in L.

The modular non-distributive lattices are known. We gave a description of non-supermodular modular lattices.

Further we characterize these lattices thus. A lattice is supermodular if and only if it is isomorphic to a subdirect union of copies of C_2 and M_3. (Where C_2 denotes the two element chain). As a corollary, we see that a lattice is supermodular if and only if its dual is supermodular.

30. Confinitely projective modules : *V.A. Hiremath.*

V,S. Ramamurthi and K.M. Rangaswamy [J. Austral. Math. Soc. 16 (1973), 243-248] have studied finitely injective modules. The author has studied finitely projective [J. Austral. Math. Soc. (Series A) 26 (1978), 330-336] and confinitely injective [London Math. Soc. (2) 17 (1978), 28-32] modules. In the same vein, we introduce in this paper, the notion of a confinitely projective module and study its properties over various types of rings. Every confinitely projective module is always torsionless. Over a (Semi) hereditary ring, every (finitely generated) cofinitely projective module of finite Goldie dimension is projective. Over an Artinian ring, every cofinitely projective module is projective. Cofinitely projective modules over a Dedekind domain are flat and reduced. In particular cofinitely projective abelian groups are studied.

31. Special faithful representation of a po-universal algebra into a po-group : *S.P. Bandyopadhyay and Manju Bhattacharyya.*

A universal algebra A is called a po-universal algebra iff it is a poset satisfying the law of monotonicity for all operations of arness not less than two. The idea of specially derived algebras has been introduced by Cohn and Rebane. It has been shown in this paper that every po–universal algebra with signature Ω, admiting no order reversible unary operator is o-monomorphic to some specially derive Ω po-algebra over some po-group.

32. Sheaves over locally Boolean spaces : *U.M. Swamy and G.C. Rao, (Waltair).*

This paper is devoted mainly to characterize the set of all global sections of a sheaf of sets over a locally Boolean space purely algebraically. An almost Boolean ring (ABR) is an algebra $(R,+,\cdot,0)$ of type (2,2,0) satisfying all the axioms of a Boolean ring except possibly the associativity of + and satisfying the equation

$$\{(x+y)+z\}\ t=\{x+(y+z)\}t.$$

Here, we prove that there is a one-to-one correspondence between the class

of ABRs and the class of sheaves of sets over locally Boolean spaces. Using this, we gave a simple equivalent condition to a question raised by Subrahmanyam in his paper 'An extension of Boolean lattice theory'. (Math. Ann., 151(1963), 332-345).

33. Birkhoff centre of a semigroup : *U. Maddana Swamy and G. Suryanarayana Murti (Waltair).*

Earlier, in an attempt to represent an arbitrary universal algebra as the algebra of all global sections of a sheaf of algebras (of the same type) over a Boolean space, we have introduced the notion of the 'Boolean centre' (A) of a universal algebra A as the set of all balanced direct factors admitting balanced direct complements and proved that (A) is a permutable Boolean sublattice of the structure lattice (A) of A. Further, we have proved that the Boolean centre $(S)$of a semigroup S with 0 and 1 is precisely the set of all direct factors. In this connection, we have also introduced the concept of 'Birkhoff centre' of a semigroup with 0 and 1 analogous to that of a bounded poset and proved that it is a Boolean algebra in which the meet operation coincides with that in S and that it is isomorphic with the Boolean centre (S) of S. In this paper, we extend the notion of this Birkhoff centre to a more general class of semigroups and prove that it is a relatively complemented distributive lattice (provided it is non-empty) and is isomorphic with a sublattice of its Boolean centre.

34. A contradiction; on minimal primes in Noetherian UFD'S : *G. Ekanathan, (Banglore).*

Let x_1, x_2 be an R-sequence in a Noetherian UFD R with the constraint that the ideals $I_1 = (x_1)$ and $I_2 = (x_1, x_2)$ are non-prime ideals of R. The present note investigates this kind of situation in detail before concluding the inevitability of the occurrence of peculiar self-contradictions within the system of study; and hence creates a new field of inquiry.

35. Birkhoff centre of semilattices : *G. Suryanarayana Murti (Bobbili).*

Let $(S, .)$ be a semigroup satisfying the following

(1) To each $x \in S$, there exists central idempotents e and f such that $ex = e, fx = x$.

(2) the set E_S of all central tdempotents of S is directed above. An element a of S is said to be central in S if and only if a belongs to the Birkhoff Centre $B(e, f)_S$ of the bounded semigroup $[e, f]_S$ for any $e \leqslant f$ and $e, f \in E_S$. In this paper we characterise a B-central element of a directed semilattice $(S, \wedge)$ by means of meet and join ditributivity of a.

II NUMBER THEORY, COMBINATORICS, CODING THEORY

36. **On some cyclic square properties of the famous number** 142857 : *D.R. Kaparekar* (*Deolali Camp*).

Numbers like $45^2=2025$, where $20+25=45$ (the original squared number) or $297^2=88209$ where $209+88=297$ (297 is the original squared number) are declared as Kaprekar numbers in *Science Today* (*December 1978 at page 51-53*). There is a long list of such numbers invented by me before 40 years (without any use of computor). You will find in the list also 142857 as one of the Kaprekar numbers because $(142857)^2=020408\ 122449$ where $122449+020408=142857$ (the original number which was squared).

If 142857 is called as $a\ b\ c\ d\ e\ f$ then $(abcdef)^2$ gives a big number where the two halves of the square added together gives again $a\ b\ c\ d\ e\ f$. Now the wonderful property to be declared today is about any cyclic arrangement of this 142857. I declare that any cycle of the number $a\ b\ c\ d\ e\ f$ that means $b\ c\ d\ e\ f\ a$, $c\ d\ e\ f\ a\ b$, $d\ e\ f\ a\ b\ c$, $e\ f\ a\ b\ c\ d$ or $f\ a\ b\ c\ d\ e$ if squared and the two halves of the square are added together then the result is some one of the 6 cyclic numbers again. Thus take cycle $=b\ c\ d\ e\ f\ a$ or $(428571)^2=183673\ 102041$ where the 2 halves $102041+183673=285714$ $=c\ d\ e\ f\ a\ b$. Thus $(bcdefa)^2$ gives $cdefab$.

Similarly $(cdefab)^2$ or $(285714)^2=081632\ 489796$

where $489796+081632=571428=efabcd$.

Similarly $(defabc)^2$ or $(857142)^2=734692\ 408164$

where $408164+734692=1142856$

adding the additional left hand 1 to right 6 we get $142857=abcdef$. Thus $(defabc)^2$ gives $abcdef$. Also $(efabcd)^2$ or $(571428)^2=326529\ 959184$ where $959184+326529=1285713=285714$ (after adding the left 1 to 3 on right) $=cdefab$ and lastly $(fabcde)^2=(714285)^2=510203\ 061225$

where $510203+061225=571428=efabcd$ or $(fabcde)^2$ gives $efabcd$. Thus any cyclic order of the number 142857 when squared and two halves added together give us some of the cyclic number of 142857. In this note this wonderful new discovery is described fully. All the Kaprekar Numbers given on page 51-53 also observed similar cyclic property for their squares. The two halves of the squared number will again give some cycle of the

original number. Readers may try any of the 200 numbers given at page 53 of Science Today. Can any one give a complete proof for this new wonderful property ? Please write to the above address.

37. **Mathematical number patterns** : *K. Suresh* (*Bangalore*).

When natural numbers, satisfying certain constraints, are subject to basic arithmeatical operations in a specific order, we get a consquent end result. This result is applicable for all numbers satisfying the constraints. Thus such a class of numbers give rise to a mathematical number pattern. The manifestation of the patterns may be occurance of arithematical progression etc.

Five such patterns have been dealt with in this paper. The first pattern deals with certain products of two digit numbers. The result is occurance of arithematical progressions (Products of type 11×11, 12×21...). Second pattern is that difference of numbers having same digital sum is a multiple of 9. Third pattern is that the product of a number having repeated digits, with any number is a Demlo number. Fourth one is that when the product of two 12321, and a four digited multiple of 9 is split and added suitably we get back the original number reversed. Fifth is that digital root of any multiple of 9 is 9 itself.

38. **Prime numbers :** *V. Balasubramania Sarma* (*Madras*).

Any number is of the form $6n$, $6n+1$, $6n+2$, $6n+3$, $6n+4$ or $6n+5$. $6n+2$, $6n+3$, $6n+4$ are composite numbers. Hence any prime number is of the form $6n+1$ or $6n+5$. If n is of the form 10^r, $6n+5$ is composite. Hence a great prime number can be written in the form 6.10^r+1.

But 6001 is divisible by 17.
60001 is divisible by 29.
600001 is divisible by 19.
6000001 is divisible by 7.
60000001 is divisible by 151.
600000001 is divisible by 509.

Therefore a great prime number is 6.10^r+1 where r is greater than 8.

39. **Uniform distribution and infinite matrix** : *G. Dass and B.K. Patel.*

The object of this paper is to introduce a generalised distribution of sequences which in a single sweep generalises the concepts like uniform

distribution of sequence., (Weyl) well distribution of sequences (Peter sen) periodic distribution of sequences and prove a theorem connecting Riemann-Stieltjes integrals with theory of generalised distribution.

40. **Extension of the Euler numbers and polynomials to several variables :** *B.D. Agrawal and Alpana Bhatnagar (Varanasi).*

The object of this paper is to furnish an extension of the generalized Euler numbers and the corresponding polynomials given by several workers Viz. Crombez, Norlünd, Nath and Prased. Moreover, the series form, recurrence relations, mixed generating relations and an integral representation for the extended numbers and polynomials have been deduced.

41. **Extension of the Bernoulli numbers and polynomials to several variables :** *B.D. Agrawal and Alpana Bhatnagar (Varanasi).*

The object of the present paper is to study such generalized Bernoulli numbers and polynomials of several variables, which embody several well known Bernoulli numbers and polynomials as well as their generalizations. Moreover, the series form, characterization, recurrence relations, mixed generating relations and an integral representation for the aforesaid polynomial set have been deduced.

42. **On a method of Eckford Cohen :** *V. Sita Ramaiah and D. Suryanarayana.*

Let k be a fixed integer $\geqslant 2$, β be a fixed number $>(k+1)^{-1}$ and g_k be a multiplicative function satisfying either (1) $| g_k(p^j)-1 | \leqslant p^{-1}$, for $1\leqslant j \leqslant k-1$; $g_k(p^k)=0$, for all primes p, or (1′) $g_k(p^j)=1$, for $1\leqslant j\leqslant k-1$; $g_k(p^k) =p^{-\beta}$, for all primes p, and (2) $| g_k(m) | \leqslant 1$. In this paper following a method of Eckford Cohen we establish an asymptotic formula for the sum $\sum_{m\leqslant x} g_k(m)$ and deduce several known asymptotic formulae as particular cases.

43. **A note on Farey Fibonacci sequence :** *K.C. Prasad (Ranchi).*

Consider the Fibonacci sequence $\{F_n : n\geqslant 0\}$ defined as $F_0=1$, $F_1=1$ and $F_m=F_{m-1}+F_{m-2}$: $m\geqslant 2$. Set $\Sigma_n=\{F_i/F_j \mid 1\leqslant i<j\leqslant n\}$. By the Farey-Fibonacci sequence $\mathfrak{F}_n$ of order n, we mean the sequence obtained on arranging the terms of Σ_n in ascending order. This sequence has been studied to a considerable extent by K. Alladi and H Gupta. In this note I give a formula for the general term T_t of $\mathfrak{F}_n$, not found before.

$$T_t = F_{i-2\,|\,k\,|\,+\delta(i,k)} \Big/ F_{n-2\,|\,k\,|\,+\delta(i,k)}$$

where
$$i = \begin{cases} [\sqrt{2t-2}] & \text{it } 2t \leqslant [\sqrt{2t-2}]([\sqrt{2t-2}]+1) \\ [\sqrt{2t-1}]+1 & \text{otherwise,} \end{cases}$$

$$K = t - \frac{i(i-1)}{2} - \left[\frac{i+1}{2}\right]$$

and
$$\delta(i,k) = \begin{cases} -1 \text{ if } i \text{ is even and } k \leqslant 0 \\ 0 \text{ if } i \text{ is odd and } k \leqslant 0 \\ 1 \text{ if } i \text{ is odd and } k > 0 \\ 2 \text{ if } i \text{ is even and } k > 0. \end{cases}$$

44. Series expansions of a class of arithmetic functions : *P.V. Krishnaiah and R. Sita Rama Chandra Rao.*

The main result of the paper is : *Theorem.* Let g, h be bounded arithmetic functions and α be an arithmetic function such that $\alpha(n) << n^\beta$ for some $\beta > 0$. Then for any positive integral k, real $\lambda > 0$, complex s with Re $s > \beta + \frac{1}{k} - \lambda$ and any bounded completely multiplicative arithmetic function f, we have

$$\sum_{d^k\delta = n} g(d)h(\delta)\alpha(d^k)\delta^s = n^s \sum_{d=1}^{\infty} \frac{c_k^{(\lambda)}(n;\, d,\, f,\, g,\, h)}{d^{k(\lambda+s)}} \left(\sum_{\delta=1}^{\infty} \frac{f(\delta)\alpha(d^k\delta^k)}{\delta^{(\lambda+s)}} \right)$$

where

$$c_k^{(\lambda)}(n;\, r,\, f,\, g,\, h) = \sum_{d^k \mid n,\, d \mid r} d^{k\lambda} g(d)\, h\left(\frac{n}{d^k}\right) \mu\left(\frac{r}{d}\right) f\left(\frac{r}{d}\right)$$

μ being the Mobius function.

Specializing this theorem, we deduce several formulae embodying series expansions of arithmetical functions. These formulae in turn contain as special cases various results due to S. Ramanujan (collected papers (1927), 179-199), R.D. Carmicherel (Proc. Lond. Math. Soc., (2) 34 (1934), 1—26) and E. Cohen (Duke Math. Jour., 26 (1959), 491-500 and ibid., 36 (1969), 659-668).

45. Lattice points in a hyper-sphere : *R. Sita Rama Chandra Rao.*

For $x > 0$ and positive integral k, let $A_k(x)$ denote the number of Lattice

points in the k-dimensional Euclidean sphere of radius $\sqrt{x}$ with centre $(0,0,\ldots,0)$. Writing $P_k(x)=A_k(x)-V_k(x)$ where $V_k(x)$ is the volume of the sphere in question, we prove that as $x\to+\infty$

(1) $P_4(x)=O\,(x\log x)$

(2) $P_4(x)=\Omega(x\log\log x)$.

Of course (1) and (2) are well-known. We give a short proof for (1). (2) is an easy consequence of

$$(3)\quad \overline{\lim_{n\to\infty}}\ \frac{r_4(n)}{n\log\log n}=6e^{\gamma}$$

where $r_4(n)=\displaystyle\sum_{\substack{x_1,x_2,x_3,x_4=-\infty\\ x_1^2+x_2^2+x_3^2+x_4^2=n}}^{\infty}1$. (3) is believed to be new.

46. **On a formula of Riordan's :** *V.L.N. Sarma (Varanasi).*

Catalan's formula

$$c_n=\frac{1}{n}\binom{2n-2}{n-1}$$

gives the number of non-associative products of n factors in prescribed order. When parentheses are used to fix the order of multiplications in a product, there appear one or more nests of parentheses. The number k of these nests satisfies the obvious inequalities : $2\leqslant 2k\leqslant n$. The formula

$$c_{n,k}=k\,2^{n-2k}\,\frac{(n-2)!}{(n-2k)!\,(k!)^2}$$

for the number of products of n factors with k nests has been found by Riordan [Amer. Math. Monthly, 80 (1973) 904-906].

By elementary arguments we show that the unique minimum of $c_{n,k}$ is at $k=[\frac{1}{2}n]$; and the unique maximum at $k=[\frac{1}{2}[\frac{1}{2}n]+\frac{1}{2}]$.

47. **A new technique for evaluation of limit points of ranges of some arithmetic functions :** *B.S.K.R. Somayajulu (Cuttack) and Hajari Sahu (Bhadrak).*

Let n denote a positive integer. Let $f\,(n)$ be a multiplicative arithmetic

function satisfying the conditions ;

(a) $0 < f(n) < 1$ for every n.

(b) $f(p_r)$ tends to 1 as r tends to ∞ where p_r denotes the r th prime,

(c) $\prod_{k=1}^{n} f(p_k)$ tends to zero as n tends to ∞.

Then the set of numbers $f(1)$, $f(2)$,... $f(n)$,...is dense in [0, 1]. This theorem is proved with the help of Somayajulu's Lemmas on sub-series of infinite series and sub-productsof infinite products. The use of this theorem as a technique for finding limit points of ranges of arithmetic functions is illustrated by many examples.

48. **Rectification of numbers :** *By A.R. Rao.*

Let two numbers n_1, n_2 in the decimal scale have each r digits and neither of them be divisible by 7. Then n_2 is defined as rectifier of n_1, if, whenever any one digit of n_1 is replaced by the digit in the corresponding place of n_2, n_1 becomes divisible by 7. We discuss in this paper properties of rectifiers and in particular, prove that for a given r, there exists a cyclic sequence n_i $(i=1,2,..,p)$ of numbers, p depending on r, such that n_{i+1} is the rectifier of n_i (the rectifier of n_p being n_1). A generalisation to numbers in seales other than 10 and modulus other than 7 is possible.

49. **A characterization of an arithmetic function with ordered unique representation set as its range :** *S. Audinarayana Moorthy* (*Cuttack*).

This is a sequel to the author's paper entitled "Unique representation sets" presented at the 65th Session of the Indian Science Congress (see Abstract No. 94).

Let $(x_n)_{n=0}^{\infty}$ be a sequence of positive real numbers such that $\Sigma_{i=0}^{n} x_i < x_{n+1}$ for every n. Then it can be shown that finite sums of *distinct* members of the sequence are all distinct. The set of all such sums is referred to as an *ordered unique representation set.*

In this paper we prove the following

Theorem. Let f be a strictly increasing arithmetic function with an ordered unique representation set as its range.

If $n=2^{t_1}+2^{t_2}+\ldots+2^{t_r}$, $t_i \neq t_j$ for $i \neq j$.

then $f(n) = x_{t_1} + \ldots + x_{t_r}$.

(Note. Every positive integer is expressible uniquely as sum of *distinct* non-negative powers of 2).

Also, we give a method of determining the number ($\gamma_k(n)$) of numbers not greater than n and expressible as sum of *distinct* powers of a positive integer $k > 1$. If m be the largest number not greater than n and expressible as

$k^{t_1} + \ldots + k^{t_r}$ (with t's all distinct), then

$$\gamma_k^{(n)} = 2^{t_1} + \ldots + 2^{t_r}.$$

50. On the order of the error function of the square full-integers. II : *D. Suryanarayana.*

Let $L(x)$ denote the number of square-full integers not exceeding x. By a square-full integer we mean a positive integer all of whose prime factors have multiplicity at least two. It is well-known that

$$L(x) \sim \frac{\zeta(3/2)}{\zeta(3)} x^{\frac{1}{2}} + \frac{\zeta(2/3)}{\zeta(2)} x^{\frac{1}{3}},$$

were $\zeta(s)$ denotes the Riemann zeta function. Let $\triangle(x)$ denote the error function in the asymptotic formula for $L(x)$. On the assumption of the Riemann hypothesis (R.H.), it is known that

$\triangle(x) = O\left(x^{\frac{13}{81}+\epsilon}\right)$ for every $\epsilon > 0$. In this paper, we prove on the assumption of R.H. that

$$\frac{1}{x}\int_1^x |\triangle(t)|\, dt = O\left(x^{\frac{1}{10}+\epsilon}\right).$$

In fact, we prove some general results and deduce the above from them. On the basis of this result, we conjecture that

$\triangle(x) = O\left(x^{\frac{1}{10}+\epsilon}\right)$. under the assumption of R.H.

51. **Perfect totient numbers :** *A.L. Mohan and D. Suryanarayana.*

M.V. Subbarao (Math. student Vol. XXIII 1955-*p*178) considered the solutions of $S(n)=\varphi_1(n)+\varphi_2(n)+ \dots +\varphi_t(n)$, where $\varphi(n)$ is the Euler Totient function, $\varphi_1(n)=\varphi(n)$, $\varphi_i(n)=\varphi\ (\varphi_{i-1}(n))$ and t is the least positive integer such that $\varphi_t(n)=1$. T. Venkataraman (Math student, Vol XLIII No. 2 (1975 *p*178) called such '*n*' as perfect Totient Numbers (PTN for short). In this paper, the following theorems that characterise PT numbers are established and the conjectures made by Subbarao and Venkataraman are examined.

(1) $3p$ is *not* a PTN if p is an odd prime of the form $4k+3$.

(2) $3^2.p$ is a PTN where $p=2.3^2(2^5.3^x+1)+1$ is a prime such that $\frac{p-1}{2.3^2}$ is also a prime.

(3) $3^3.p$ is a PTN whenever $p=2^6.3^\alpha+5$ is a prime such that $\frac{p-1}{4}$ is also a prime.

(4) $F_r.\ (2^\alpha.F_{r+1}+1)^2$ is a PTN (where $F_r=2^{2^r}+1$ is a Fermat prime and $2^\alpha\ F_{r+1}+1$ is a prime) if and only if $r\neq0$ and $\alpha=1$.

(5) $F_{r+1}^{4}\ .p$ is a PTN where $F_r=2^{2^r}+1$ is a Fermat prime and $p=2^\alpha.F_r+1$ is a prime) iff $r=0$ and $\alpha=1$.

(6) $F_r{\cdot}p$ (where $F_r=2^{2^r}+1$ is a prime and $p=2^\alpha.m+1$ is a prime where m is a PTN) is a PTN iff $r=0$ and $\alpha=1$ or $\alpha=r=m=1$.

52. **Phasing traffic signals at Kanpur :** *M.R. Sridharan, and Merajuddin (Kanpur).*

A solution for the optimum waiting time for drivers of vehicles at Parade Choraha, Mall Road, Kanpur, an important intersection, has been obtained by using a characterization of interval graphs. The total waiting time for drivers is considerably reduced by implementing our suggestions.

53. **A reversible code and its characterization :** *Bal Krishan Dass, Sunil Kumar Muttoo, (New Delhi).*

The paper deals with the construction of codes developed from a matrix obtained by annexing two triangular matrices, one lower triangular and the other upper triangular, such that the last column of the first is the first column of the second. The codes that are null spaces of these circulant matrices are shown to be reversible. It is also shown that dual of such

codes is also reversible. Further, it is shown that such codes have minimum weight at least 3 and are capable of correcting well defined class of solid bursts of odd length. The paper is concluded with an example.

54. **Orthogonality in matroids and Welsh's conjecture :** *R. Balakrishnan, N. Sudharsanam, (Tiruchirapalli).*

Fix any base B of a matroid on a finite set S. For $x \varepsilon S$, let $D(x)$ denote the unique minimal subset of B on which x depends. For $x, y \varepsilon S$, set x orthogonal to y iff $D(x) \cap D(y) = \phi$. While studying certain properties of this orthogonolity, *D.J.A.* Welsh raised the following conjecture : If x is orthogonal to an independent subset A of S and if y depends on A, then x is orthogonal to y. In this paper, we prove this conjecture in the stronger form when A is any subset of S.

55. **Powers of chordal graphs:** *R. Balkrishnan and P. Paulraja (Tiruchirapalli).*

An undirected simple graph G is called chordal if every cycle C_k of G of lenght $k>3$ has a chord. For a chordal graph G, we prove the following : (i) If m is an odd positive integer, G^m is also chordal. (ii) If m is an even positive integer and if G^m is not chordal then none of the edges of any chordless cycle of G^m is an edge of G^r, $r<m$.

56. **Space of cycle mappings of a graph :** *R. Baklrishnan, N. Sudharsanam (Tiruchirapalli).*

A real valued function on the line set of a graph G is called a cycle mapping of G if it vanishes on each cycle of G. This paper examines the structure of the real vector space of all cycle mappings of a graph G. The dimension of this vector space is the cocycle rank of G. Looking at this dimension from two totally different angles, it is shown that zero mapping is the only cycle mapping of G iff G is 3-line connected.

57. **Some contributions on matchings and independent sets in a graph :** *I. H. Naga Raja Rao and R. Sudhakar (Waltair).*

The concepts of maximality of Matchings and Independent sets in a graph are introducted with respect to the (order) number of elements of these sets (see Graph theory with Applications, J.A. Bondy and U.S.R. Murty).

Now, we introduce the analogous concepts with respect to the set inclusion relations and observe that these concepts are extensions of the order concepts. As an illustration we exhibit a simple graph that admits a set maximal matching which is not an order maximal matching. Some interesting results on complete graphs have been also observed,

58. **Pseudo self-complementary trees :** *Sudhir Kumar Shukla* (*Kanpur*).

A graph G is said to be direct pseudo self-complementary (D.P.S.C.) if there is a spanning subgraph of $\bar{G}$ which is isomorphic to G. G is reverse pseudo self-complementary (R.P.S.C.) if $\bar{G}$ is a D.P.S.C. graph. It is easy to check that no tree with more than four vertices is a R.P.S.C. graph. In this paper, we prove that a tree is a D.P.S.C. graph if and only if it is not a star.

59. **Graphs which are switching equivalent to their iterated line graphs ; I. Connected case :** *B. D. Acharya* (*Allahabad*).

In this paper, the problem of determining graphs which are switching equivalent to at least one of their iterated line graphs is considered, and such connected graphs are characterized.

60. **Withdrawn**

III TOPOLOGY

61. **R^3 cannot be foliated by circles :** *A. K. Vijayakumar* (*Madurai*).

Epstein has asked (Differential Topology, Foliations and Gelfand-Fuks Cohomology, Springer-Verlag Lecture Notes No. 652, page 252) whether R^3 can be foliated by circles. This paper provides a negative answer.

62. **Neighboform closure :** *R. N. Lal*, (*Bhagalpur*).

Our neighboform poset $<P, V>$ is a poset P with a set $V=\{u : M \to P \mid M \subset P\}$ of mappings satisfying *nhd* like properties (Cal. Math. Soc. 1977). If P is a complete lattice then M-adic closure and interior are defined by setting :

$$\bar{a} = \wedge u(a), \ \underline{a} = Vx \in M \mid \exists u \ s.t. \ u(x) < a.$$

In such a *nfm* lattice M-adic seperation is also introduced including proximal and uniform seperation of elements. (J. of App. and Pure Math. Soc. 1975).

63. **Games in a scattered space :** *R. N. Lal* (*Bhagalpur*) *and B.P. Kumar* (*Patna*).

R. Telgarsky has studied topological properties defined by games

(1972). The paper is concerned with the determination of games on a scattered space. The main result of the paper is that the game is determined on (*i*) a scattered and lindelof space and (*ii*) a scattered and para-compact space.

64. Bitopological dynamics : *R.N. Lal (Bhagalpur) and M.K. Saha (Tripura).*

Our bitopological dynamics is a triple

$$< <X, \tau_1, \tau_2>, <T, \sigma_1, \sigma_2>, \pi>$$

consisting of a bitopological space $<X, \tau_1, \tau_2>$, a bitopological group $<T, \sigma_1, \sigma_2>$ and a mapping $\pi : <X, \tau_1, \tau_2> \times <T, \sigma_1, \sigma_2> \rightarrow <X, \tau_1, \tau_2>$ satisfying identity and homomorphism axioms as well as bicontinuity axiom *i. e.*, π is bicontinuous on $<X, \tau_1, \tau_2> \times <T, \sigma_1, \sigma_2>$.

As a bicontinuous mapping is pairwise continuous, a bitopological dynamics is a pairwise topological dynamics whose properties have been studied.

65. M-adic connectedness on a nfm lattices : *R.N. Lal (Bhagalpur).*

An element a in a nfm lattice is M-adic disconnected provided $a = x_1 \cup x_2$, $x_1 \underset{m}{H} x_2$, otherwise connected, which includes the notion of uniform and proximal connectedness (Proc. Nat. Acad. Sciences 1978). The notion of connectedness can be extended to that of connectedness between two elements leading to the notion of quasi components. For a nhd atomistic lattice local connectivity is also introduced.

66. Connectedness characterization p→q→r→∼p : *G. Ekanathan (Bangalore).*

The possibility of deducing a concept to an equivalent form of the type $p \rightarrow q \rightarrow r \rightarrow \sim p$ has been evidenced by the following proven results :

(*i*) A Topological Space (X, τ) is a dense-in-itself space iff $d(E) = \phi \Rightarrow E^\circ = \phi$ for any non-void subset E of X.

(*ii*) A dense-in-itself T_1-space (X, τ) is connected iff $d(E) \cap d(E^c) = \phi \Rightarrow d(E) = \phi$ or $d(E^c) = \phi$ for any non-void proper subset E of X.

(*iii*) A T_1-Space (X, τ) is connected iff $b(E) = \phi \Rightarrow E^\circ \neq \phi$ and $E_c^\circ \neq \phi \Rightarrow d(E) \neq \phi$ and $d(E^c) \neq \phi \Rightarrow d(E) \cap d(E^c) \neq \phi \Rightarrow b(E) \neq \phi$ for any non-void proper subset E of X iff $p \rightarrow q \rightarrow r \rightarrow \sim p$.

67. **Syntopogeneous connectedness between sets :** *R. N. Lal and S.K. Singh (Bhagalpur).*

Sieber and Pervin have introduced the notion of connectecness in a syntopogeneous space X which can be generalised to that of connectedness between two sets A and B that is iff, there does not exist a separation

$$X=X_1 \cup X_2,\ X_1 \mid\!-\!\mid X_2$$

where $A \subset X_1$ and $B \subset X_2$ leading to the notion of quasi-components. Properties of such connectedness and quasi-components have been fully developed.

68. **On pairwise path-connectedness in bitopological spaces :** *S. M. Felix and K. Chandrasekhara Rao (Palayamkottai).*

Define two toplogies on [0, 1] to make it a bitopological space. If (X, τ_1, τ_2) is a bitopological space, a *path* in X from points x to y, $x, y \in X$, is a pairwise continuous map $f: [0, 1] \rightarrow X$ such that $f(0)=x$, $f(1)=y$. X is *pairwise path-connected* if every pair of points of X can be joined by a path in X. The following two results are established : *Theorem 1.* A pairwise path-connected space is necessarily pairwise connected. *Theorem 2.* (X, τ_1, τ_2) is pairwise path-connected iff (X, τ_1) and (X, τ_2) are path-connected.

69. **On pairwise nearly compact bitopological spaces :** *S.M. Felix and K. Chandrasekhara Rao (Palayamkottai).*

A set P of a bitopoloyical space (X, τ_1, τ_2) is said to be *δ-closed w. r. to* τ_1 if for each point $x \notin P$, $\exists$ a τ_1-open set G containing x such that $\tau_1\text{—}int\tau_2\text{—}clG \cap P=\phi$. A set G is δ-open *w. r.* to τ_1 if its complement is δ-closed *w. r.* to τ_1. The topology formed by δ-open *w. r.* to τ_1 sets is denoted by τ_1^* and $\tau_1^* \subseteq \tau_1$. (X, τ_1, τ_2) is *pairwise nearly compact* if every $\tau_1\ \tau_2$-open cover $\{C_k\}$ has a finite subfamily the τ_i-*int* τ_j-closures of whose members cover X, $i, j,=1, 2$; $i \neq j$. A function $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is *pairwise almost continuous* iff the induced functions $f: (X, \sigma_i) \rightarrow (Y, \sigma_i)$, $i=1, 2$, are almost continuous. The following results are established : *Theorem 1.* (X, τ_1, τ_2) is pairwise nearly compat iff (X, τ_1^*, τ_2^*) is pairwise compact. *Theorem 2.* A bitopological space X is pairwise nearly compact iff it is a pairwise almost continuous Image of a pairwise compact space. *Theorem 3.* In (X, τ_1, τ_2), if every $\tau_1\ \tau_2$-open subset containing a pairwise dense subset A contains a pairwise nearly compact set containing A, then A is pairwise nearly compact. *Theorem 4.*

In (X, τ_1, τ_2), if every pairwise open subspace is pairwise nearly compact, then every pairwise dense subspace of X is pairwise nearly compact. *Corollary.* Finite union of pairwise nearly compact sets, which are either open or pairwise dense, is pairwise nearly compact.

70. On the completion of a metrizable topological vector space of functions : *Harjeet Singh and Dinesh Chandra Jha* (*Darbhanga*).

In this paper an attempt has been made to prove the results on the completion of a metrizable topological vector space of functions.

71. Strong continuity and strong compactness : *B. M. Munshi and D.S. Bassan.*

The paper consists of two parts. In part one we have defined a function $f: X \to Y$ (X and Y are topological spaces) to be strong continuous if inverse image of a semi open subset of Y is an open subset of X. We have found several properties of such functions and have compared them with irresolute function, continuous functions and semi-continuous functions.

In part two we have introduced a concept of strong compactness by replacing open sets by semi open sets in the usual definition of compactness and have studied quite a few interesting properties of such spaces. We have also studied how image of strong compact spaces behave under irresolute, continuous and semi-continuous mapping.

72. Semi-mesocompact spaces : *D. K. Thakkar and Darsan Singh,* (*Ahmedabad*).

Semi-mesocompactness is a generalization of paracompactness. It is proved that (*i*) A normal space X is mesocompact if and only if X is semi-mesocompact and countably subparacompact. (*ii*) If a space X is compact-expandable and if every open covering of X has a σ-locally-finite refinement then X is mesocompact. (*iii*) A compact-expandable developable space is semi-mesocompact and hence metrizale. (*iv*) In a collectionwise normal, k-space, the notions of paracompactness, semi-paracompactness, semi-mesocompactness + countable paracompactness and mesocompactness are equivalent.

73. On lattices of compactifications of a product : *C. P. Padmaja* (*Cochin*).

$K(Z)$ denotes the complete lattice of all Hausdorff compactifications of a locally compact Hausdorff space Z. In this paper we establish an isomorphism from the product lattice $K(X) \times K(Y)$ into $K(X \times Y)$. This also settles the problem for arbitrary products of spaces. Analogous results are obtained for the semilattice of compactifications when the spaces are not necessarily locally compact.

74. A note on Wallman compactification : *Asha Rani Singal and Sunder Lal.*

For a T_1 space X, let X^* denote the Wallman compactification consisting of all ultra-closed filters on X. It is known that X is T_4 iff X^* is T_2. In this note we introduce two weaker forms of T_2 axiom viz., relatively T_2 and relatively completely T_2 axioms and prove (*i*) X is T_3 iff X^* is T_2 relative to X ; and (*ii*) X is $T_{3\cdot 5}$ iff X^* is completely T_2 relative to X.

75. The uniqueness of the compact group topology of the p-adic integers : *S. Janakiraman (Madurai).*

In this note, we supply a straight forward proof of the uniqueness of the compact group topology of the p-adic integers I_p and also that of $I_p^n \times I_q^m$ for $p \neq q$, n, m positive integers.

76. Fixed points of multivalued mappings in uniform spaces : *S. N. Mishra (Garhwal), S. L. Singh (Dehra Dun).*

For a multimapping of a uniform space, notions of contraction, nonexpansion and asymptotic regularity are introduced. Using the concept of α^*-measure of noncompactness for a nonempty P^*-bounded subset of a uniform space, the notion of a condensing multimapping is also introduced. These ideas are used to obtain several results on fixed points of contractive, nonexpansive, asymptotically regular and condensing multimappings in a uniform space.

77. Fixed points of commutng mappings : *Chitra Kulshrestha (Dehra Dun), B.M.L. Tiwari (Chamoli, U.P.).*

The aim of this paper is to improve the main results of Yeh (1979, Indian J. pure appl. Math., 10 (4) : 415-420). The condition that all the three mappings satisfying the functional condition be continuous in Yeh, op. cit., is relaxed considerably by requiring the continuity of only one mapping which appears only on the right hand side of the functional con-

dition. Moreover, the main result is further extended to non-complete metric spaces.

78. Common fixed points of commuting mappings in 2-metric spaces : *S.L. Singh (Dehra Dun), B. Ram (Tehri Garhwal).*

The purpose of this paper is to give some common fixed point theorems for four self-mappings P, Q, S and T of a 2-metric space (X, d) satisfying

$$d\,(P_x, Q_y, a) \leqslant h.\max\{d(S_x, T_y, a), d(P_x, S_x, a), d(Q_y, T_y\, a),$$

$$\tfrac{1}{2}\,[d\,(P_x, T_y, a) + d\,(Q_y, S_x, a)\,]\,\}$$

for all x, y, a in X. As a consequence, a multitude of fixed point theorems may be obtained as corollaries.

79. Fixed point theorems for strongly nonperiodic families of mappings: *P. Srivastava and S. C. Srivastava (Allahabad).*

For a single self mapping of a topological space X, the notions of being strongly nonperiodic and of being orbitally continuous were defined by Lj. B. Ciric. He has proved a fixed point theorem for a strongly nonperiodic orbitally continuous map. In the present paper, definitions of a strongly nonperiodic family and of an orbitally continuous family of self mappings of a topological space have been introduced and common fixed point theorems for a strongly nonperiodic family of self mappings of a topological space have been obtained. These extend a theorem of Ciric for a single self mapping and generalise theorems of M. T. Kiang for a commutative semigroup of self mappings of a metric space.

80. A note on fixed points in 2 metric spaces : *Ashok Kumar (Delhi).*

The note concerns with some results on fixed points of mappings on 2-metric spaces satisfying conditions of the contractive type.

81. An extension of Caristi's theorem to multifunctions : *J. Madhusudana Rao.*

In this note we have proved the following result : Let (M, d) be a complete metric space. Let S be a nonempty subset of $CB\,(M)$, the set of all nonempty closed and bounded subset of M. Assmue that $\{x\}$ is in S for all x in M. Let S be equipped with the Hausdorff metric h. Let $g : M \rightarrow S$ be an arbitrary function and ϕ, a lower semi continuous function of S into the set of positive real numbers. If

(*i*) A, B are in S and $A \subseteq B$ imply $\phi\,(A) \leqslant \phi\,(B)$ and
(*ii*) $h\,(\,x, g\,(x)\,) \leqslant \phi\,(x) - \phi\,(g\,(x)\,)$, then g has a fixed point.

As a corollary to the above results, we obtain a famous fixed point theorem due to Caristi.

Finally, we give an example to show that condition (i) is essential for the conclusion of the above result.

82. **Anilian 'A' spaces :** *Anil Kumar Baburao Satpute* (*Bombay*).

Our interest is to define the sequential topology which has some particularity. The members of the topology "**T**" are some what arranged.

83. **Anilian 'B'** *spaces* : *Anil Kumar Baburao Satpute* (*Bombay*).

Here, we define a new topology which has some particularity.

84. **$\in$—Ring-space :** *Anil Kumar Baburao Satpute* (*Bombay*).

Let X be a topological space. Y is any open subset of X. Then Y^c will be closed subset of X. If we consider sufficiently small positive number. Consider the set of all points such that, open set Y is increased by $\in$. Then this new Y denoted by Y' makes Y as closed subset of X. $Y'=(Y+\in)$. Then $(Y'-\in)^c$ is open.

Then the space which is made by the displacement of Sub Y to Y' by $\in$ is called $\in$—*Ring*. All points belonging to this particular set is called $\in$—Ring Space.

IV FUNCTIONAL ANALYSIS, HARMONIC ANALYSIS

85. **A generalization of the notion of Weyl's spectrum :** *B. S. Yadav and S. C. Arora.*

If H is a Hilbert space of dimension $h \geqslant N_0$, the cardinality of the set of natural numbers, then for each cardinal α, $N_0 \leqslant \alpha \leqslant h$, the notion of the Weyl's spectrum of weight α, $w_\alpha(T)$, is introduced and studied. Correspondingly, α-Weyl theorem is introduced and it is proved that a class of normal operators satisfies α-Weyl theorem.

86. **Mapping theorem for Weyl spectrum of isoloid operators :** *N. N. Chourasia and P. B. Ramanujan* (*Rajkot*).

For a bounded linear operator T on a Banach space, let f be a function analytic in a neighbourhood of $\sigma(T)$, the spectrum of T and let $w(T)$ denote the Weyl spectrum given by

$$w(T)=\cap\ \{\sigma(T+K) : K \text{ a compact operator}\}.$$

One says that Weyl's theorem holds for T if $w(T)$ consists of $\sigma(T)$ except for the isolated eigenvalues of finite (geometric) multiplicity. T is said to

be isoloid if every isoloid point of $\sigma(T)$ is an eigenvalue of T. In this paper, we prove the following

Theorem. Let T be isoloid. Then the relation

$$w(f(T)) = f(w(T))$$

holds if and only if Weyl's theorem holds for $f(T)$.

Corollary. If T is a hyponormal operator (that is, $T^*T \geqslant TT^*$) in a Hilbert space, then Weyl's theorem holds for $f(T)$.

This answers, in particular, the question "for a hyponormal T, does Weyl's theorem hold for T^2?" raised by K.K. Oberoi in his paper. "On Weyl's Theorem II". Illinois J. Math. 21 (1977), 84–90.

87. **Characterizations and invariant subspaces of composition operators :** *D. K. Gupta and B. S. Komal* (*Jammu*).

The composition operator C_T on a weighted sequence space l^2_w induced by the mapping T from the set N of natural numbers into itself is a bounded linear transformation defined by $C_T f = f \circ T$. In this paper we have given several characterisations for a bounded operator to be a composition operator. It is also found that every composition operator on a weighted sequence space has non-trivial invariant subspace. The main theorems of the paper are

Theorem 1. Let $A \in B(l^2_w)$. Then A is a composition operator if and only if for every $n \in N$, there exists an $m \in N$ such that $A^* e'_n = e'_m$, where $e'_n = e_n / w_n$.

Theorem 2. Let $A \in B(l^2_w)$. Then A is a composition operator if and only if there exists a partition $\{E_n\}$ of N such that $Ae_n = X_{E_n}$, where X_E denotes the characteristic function of this set E.

Theorem 3. Let $C_T \in B(l^2_w)$. Then C_T has a non-trivial invariant subspace.

88. **On transloid operators :** *Madhavi Pandya and I.H. Sheth* (*Ahmedabad*).

We answer negatively the following question raised by T. Saitô :

Does relation $E(T) \cap W(T) \subseteq \sigma_p(T)$ holds for transloid operators ? (where $W(T)$, $E(T)$ and $\sigma_p(T)$ stand for the numerical range, set of extreme points of the closure of $W(T)$ and the point spectrum respectively, of an operator T defined on a Hilbert Space H).

89. Tensor products of hyponormal operators and joint spectra : *A.B.Patel (Vallabh Vidyanagar).*

Brown and Pearcy have proved that the spectrum of the tensor products of operators is the product of their spectra. We prove an analogous result for the tensor products of n-tuples of commuting hyponormal operators.

90. On Rota's models for operators : *B.S. Yadav and Ranjana Bansal.*

The paper is devoted to a construction of Rota type models for linear operators in terms of weighted shifts :

Theorem 1. Every bounded operator on a Hilbert space with spectral radius less than 1, is similar to a part of the backward unilateral weighted shift S_α with weight sequence

$$\alpha=\{\alpha_n\}_{n=1}^{\infty}$$

satisfying either of the following conditions :

(*i*) $\text{Inf}\,\{\beta_n : \beta_o=1,\ \beta_n=\alpha_1\,\alpha_2 \cdots \alpha_n \text{ if } n\geqslant 1\}>0,$

(*ii*) $$\sum_{n=0}^{\infty} \beta_n^{-2p} < \infty,\ 1 < p < \infty .$$

Corollary (*Rota's Theorem*). Every bounded linear operator on a Hilbert space with spectral radius less than 1, is similar to a part of the backward unilateral shift.

Theorem 2. Every power bounded linear operator on a Hilbert space is similar to a part of the backward unilateral weighted shift S_α with

$$\sum_{n=0}^{\infty} \beta_n^{-2} < \infty.$$

91. Spectrum of a normal composition operator on $L^2(\lambda)$: *R.K. Singh and T. Veluchamy (Jammu).*

Let (X, S, λ) be a sigma-finite measure space and let T be a non-singular transformation from X into itself. Then the mapping C_T on $L^p(\lambda)$ which takes f into $f \circ T$ is a linear transformation. If C_T is a bounded operator on $L^p(\lambda)$, then we call it a composition operator induced by T.

In this short note the following results are established :

Theorem 1. Let C_T be a normal composition operator on $L^2(\lambda)$. Then, spectrum of C_T, $\sigma(C_T) \subseteq \{\alpha : \alpha \in C$ and $|\alpha|^2 \in E(f_o)\}$ where $E(f_o)$ is the essential range of f_o, the Radon Nikodym derivative of the measure λT^{-1} with respect to the measure λ.

Theorem 2. If C_T is a non-identity hermitian composition operator on $L^2(\lambda)$, then $\sigma(C_T)=\{-1, 1\}$.

Theorem 3. Let C_T be a normal composition operator on $L^2(\lambda)$ and let $1 \notin E(f_o)$. Then, $\sigma(C_T)=\{\alpha : \alpha \in C$ and $|\alpha|^2 \in E(f_o)\}$.

92. Fixed point theorems for nonlinear random operators on Hilbert space-II : *S.V.Krishna, (Waltair).*

In the paper of the same title I, we studied fixed points for nonlinear random operators on a Hilbert space which are contractive. We introduced a generalized concept of a fixed point for such operators and proved the existence of such fixed points. In this paper we consider operators of more general nature, We discuss again existence of fixed points for pseudo contratctive and generalized contractive random operator. We also study the convergence of iterates of these operators and θ-fixed points for $0<\theta<1$.

93. Simultaneous algebraic extensions for function algebras : *R. D. Mehta and M. H. Vasavada (Vallabh Vidyanagar).*

Let A be a function algebra on X and K be a closed restriction set for A. An algebra homomorphism $T:A/_K \to A$ is said to be a simultaneous algebraic extension if $T_1=1$ and $Tf/_K=f \;\forall\; f \in A/_K$. If there exists a simultaneous algebraic extension T from $A/_K$ into A then K is called an *a.e.* set for A. In this paper we have proved that (*i*) if K is an *a.e.* set then $\hat{K}$. the A-hull of K, is a retract of $m(A)$; (*ii*) if K is a retract of X which contains the essential set for A then K is an *a.e.* set for A. We also give an example to show that an *a.e.* set, which is a retract may not contain the esential set.

94. On closed ideals of $C^{(n)}[0, 1]$: *V. D. Pathak.*

We denote by $C^{(n)}(I)$, the Banach algebra of n times continuously differentiable complex valued functions on I with the norm defined by

$$\|f\| = \|f\|_\infty + \frac{\|f'\|_\infty}{1!} + \dots + \frac{\|f^{(n)}\|_\infty}{n!}, \quad f \in C^{(n)}(I), (I=[0,1]).$$

In this paper we characterize the closed ideals of $C^{(n)}(I)$ in terms of a lower semi-continuous function from I onto the discrete space $\{0, 1, 2, \ldots, n+1\}$ and also in terms of decreasing sequences of closed subsets of I with certain properties. The later characterization is analogous to a result of Reid.

95. **Primary Banach spaces of continuous functions** : *V. Kannan and S. Radhakrishneswara Raju, (Hyderabad).*

A Banach space is said to be primary (isometrlcally), if whenever it is the direct sum of two of its subspaces, it is isometrically isomorphic to one of them.

Let n be any positive integer.

We prove that among the Banach spaces of the form $BC(S)$ where S is a countable complete metric spece with derived length n, there are only finitely many types of primary Banaeh spaces, even though there are infinitely many non-primary ones.

This finite quantity is computed in certain cases.

96. **A note on boundedly complete decompositions of a Banach space** : *P.K. Jain and Khalil Ahmad, (Delhi).*

Let (M_i) be a Schauder decomposition of a Banach space E. Then, it has been proved that the following statements are equivalent :

(A) For each number $\lambda>0$, there exists a number $r_\lambda>0$,

$$\text{such that } \left\| \sum_{i=1}^{n} x_i \right\| = 1, \quad \left\| \sum_{i=n+1}^{\infty} x_i \right\| \geqslant \lambda \quad \text{imply}$$

$$\left\| \sum_{i=1}^{\infty} x_i \right\| \geqslant 1 + r_\lambda, \quad x_i \in M_i \quad \text{for each } i.$$

(B) For every $\varepsilon>0$, there exists a $\delta>0$, such that

$$\left\| \sum_{i=1}^{n} x_i \right\| > 1-\delta, \ \left\| \sum_{i=1}^{\infty} x_i \right\| = 1 \text{ imply } \left\| \sum_{i=n+1}^{\infty} x_i \right\| \leqslant \varepsilon,$$

$x_i \in M_i$ for each i.

The main results of the paper are

Theorem 1. Let (M_i) be a Schauder decomposition of E. If (M_i) satisfies property A (or B), then (M_i) is boundedly complete. The converse may not be true.

Theorem 2. If (M_i) is a monotone decomposition of a uniformly convex space E, then (M_i) satisfies property B (hence property A).

Corollary. If (M_i) is a monotone decomposition of a uniformly convex space E, then (M_i) is boundedly complete.

97. Completion of the locally convex space and different duals : *Rohtash Kumar.*

Let $E[\tau]$ be a locally convex space with completion $\tilde{E}$ and tcpologioal dual E'. We know that $\tau_s(E')$ and $\tau_k(E')$ respectively the weak and Mackey topologies are the coaresest and the finest locally convex topologies on E for which duals are identical to E'. Similarly one can think of the coarsest and the finest locally con vex topologies on E for which completions are identical to $\tilde{E}$. In this note we haye made an attempt to answer this question. We also obtain a result (theorem 2) which exhibits the interrelationship of the three, the completion of a locally convex space, dual of a bornological space and the dual of a convergence space.

98. λ-Bases add application : *P.K. Kamthan, Manjul Gupta and M.A. Sofi, (Kanpur).*

It is well known in the theory of nuclear locally convex spaces that a Frechet space X with an absolute Schauder base $\{x_n ; f_n\}$, is nuclear if and only if $\{f_n ; jx_n\}$ is an absolute base for X^*_β. In the present paper it has been showd that the natural analogue of this situation breaks down in the case of 'λ–bases', which generalize various notions of 'absolute bases' known from the recent literature. More precisely, it turns out that the 'λ-nuclearity' of a Frechet space E with a 'λ-absolute base' is no longer implied by the λ-absolute character of its dual base in E^*_β. An example to this effect is constructed in the last section of this paper.

This paper is comprised of five sections. After summing up the introduction and the basic background concerning nuclear spaces [of. Nuclear locally convex spaces by *A.* Pietsch, Springer verlag] and sequence spaces [cf. Seqüence Spaces and Series by P. K. Kamthan and Manjul Gupta, Lecture Notes, 65 Marcel Deker, New York] in the first two sections, several notions of λ–bases are defined in section 3 and their main properties derived. The results concerning the impact of a 'λ–base' on the structure of the underlying locally convex space are also obtained in the same section, whereas in section 4, a particular type of 'λ–bases' has been singled out for further investigations. The paper concludes with section 5 which deals with the relationship of 'λ–nuclearity' with a 'λ-base',

99. Invertible and Fredholm composition operators on weighted sequence space : *D.K. Gupta, (Jammu).*

Let N denote the set of non-zero positive integers and T be a mapping from N into itself ; then the composition transformation C_T on the weighted l^2 space with weights a^n ($n \in N$ ond $a \neq 1$) defined by $C_T\, f = f\, o\, T$ is a bounded linear operator. In this paper we have characterised invertible and Fredholm composition operators on this weight sequence space.

Theorem 1. Let $1 < a < \infty$ and $C_T \in B(l_a^2)$. Then C_T is invertible if and only if T is invertible and $T(n) \leqslant k+n$ for some $k>0$ and for all $n \in N$.

Theorem 2. Let $0 < a < 1$ and let $C_T \in B(l_a^2)$. Then C_T is Fredholm if and only if the range of T contains all but finitely many elements, the restriction of T to complement of some finite set E is injective and there exists $k>0$ such that $T_1^{-1}(n) \leqslant k+n$ for all $n \in N$ where T_1 is the restriction of T to complement of E.

Theorem 3. Let $1 < a < \infty$ and $C_T \in B(l^2{}_a)$. Then C_T is Fredholm if and only if the range of T contains all but finitely many points, the restriction of T to the complement of some finite set E is injective and there exists a $k>0$ such that $T_1(n) \leqslant k+n$ for every $n \in N$, where T_1 is restriction of T to complement of E.

100. Duals of generalized sequence spaces : *Manjul Gupta, P. K. Kamthan and John Patterson (Kanpur).*

This paper mainly incorporates the study of various duals of a vector-valued sequence space (VVSS): Indeed, having introduced the notions of monotone and symetric VVSS, we establish relationship among α-, β-, γ-and δ-duals of a VVSS$\wedge(X)$. Analogous to different notions in a scalar valued sequence space (cf. Kamthan and Gupta, 'Sequence Spaces and Series', Lecture Notes, No. 65, Marcel Dakker, Inc., New York, 1980-81), we introduce various types of VVSS and prove the sequential representation of continuous and sequentially continuous linear functionals on a *VVSS*. Precisely, main results in this direction are

Proposition 1. If $(\wedge(X), \mathfrak{z})$ is a barrelled *GAK*-and *GC*-space then $[\wedge(X)]^* = \wedge^{\beta}(X^*)$.

Proposition 2. Let $(\wedge(X), \mathfrak{z})$ be a *GAK*-and *GSC*-space such that every $\sigma([\wedge(X)]^+, \wedge(X))$-bounded sequence in $[\wedge(X)]^+$ is $\mathfrak{z}$-limited.

Then $[\wedge(X)]^+ = \wedge^{\beta}(X^+)$.

Finally, we prove results related to μ-dual and $\sigma\mu$-topology on a *VVSS*

$\wedge(X)$ which we introduce corresponding to a sequence space μ. An intresting outcome of this is

Theorem 3. Let $<X,Y>$ be a dual pair such that $(Y, \sigma(Y,X))$ is sequentially complete. If $\wedge(X)$ is normal, then every $\eta(\wedge(X), \wedge^{\times}(Y))$-bounded set is $\beta(\wedge(X), \wedge^{\times}(Y))$-bounded. In particular, $<\wedge(X), \wedge^{\times}(Y)>$ is an M-system.

101. Matrix transformations on generalized sequence spaces : *Manju Gupta and J. Patterson (Kanpur).*

This paper essentially deals with the representation problem of an arbitrary continuous linear operator on vector valued sequence spaces (*VVSS*) in terms of an infinite matrix of linear operators on the underlying spaces-a study motivated by its counterpart for the scalar case, the details of which related to their importance and applications are to be found in a recent monograph of Kamthan and Cupta : Sequence Spaces aud Series ; Lecure Notes, No. 65, Marcel Dekker, Inc., New York, 1980-81.

Precisely, having introduced the notion of a matrix transformation, we prove

Proposition 1. Let $(\wedge(x), \mathfrak{J})$ and $(\wedge(Y), \mathfrak{J}')$ be two monotone barrelled GC-and GAK-spaces. Then a linear map z : $\wedge(X)\rightarrow\wedge(Y)$ is $\mathfrak{J}$-$\mathfrak{J}'$ continuous if and only if it can be represented by a matrix $[z_{ij}]$ of T_x-T_y continuous linear maps z_i^j from X to Y, where (X, T_X) and $Y, T_Y)$ are two locally ccnvex spaces with respective duals X^* and Y^*.

We also introduce the notion of transpose of a matrix of linear maps and prove a result which includes an earlier result of Allen for the scalar case (cf. the above monograph, Proposition 4.3.3).

In the final section of this paper, we consider the nuclearity and precompactness of diagonal operators and prove

Proposition 2. Let λ be a BK–AK-scquence space and X, Y two Banach spaces. If $z=\{z_i\}$ is a diagonal nuclear map from $\lambda(X)$ to $\lambda(Y)$, then each $z_i : X\rightarrow Y$ is nuclear and $\sum\limits_{i\geqslant 1} N(z_i)<\infty$. In addition, if the sequence $\{e^i\}$ of unit vectors is bounded in both the spaces λ and $\lambda^{\times}$, then converse also holds.

Proposition 3. Assuming the hypothesis of Proposition 2, if norm of each e^i is unity in λ, then z is precompact if and only if each z_i is precompact and $\| z_i \| \rightarrow 0$, as $i\rightarrow\infty$.

102. The behaviour of transformations on sequence spaces : *P. K. Kamthan (Kanpur).*

The study of different types of behaviour of a linear transformation from a sequence space λ into another sequence space μ may be considered as an outgrowth to the development of the overall theory of δ-nuclear spaces. We have attempted this study in this paper ; indeed, we consider transformations (matrix or otherwtse) from sequence spaces to themselves and consider their behaviour: More specifically, we show

Theorem 1. Let λ and μ be sequence spaces, μ being normal. Let T : $[a^i_j]$: $\lambda \rightarrow \mu$ be a matrix transformation. Then T is simply bounded and only if there exists y in $\lambda^\times$, $y>0$ such that

$$\{(\sup_j[\ |\ a_{ij}\ |\ /y_j])_i\} \in \mu.$$

Theorem 2. Let λ and μ be two sequence spaces, λ being monotone, δ a simple sequence space with $\delta = \mu^{\times\times}$. Suppose $T \equiv [a_{ij}]$: $\lambda \rightarrow \mu$ is a matrix transformation. Then T is δ-nuclear if and only if

$$\{(\sup[\ |\ a_{ij}\ |\ /y_j])_i\} \in \delta.$$

for some y in $\lambda^\times$, $y>0$.

For several definitions and results from the sequence space theory used in this paper, the reader is referred to "Sequence Spaces and Series" by Kamthan and Gupta, Marcel Dekker, Inc., Lecture Notes No. 65, New York, 1980-81.

103. On the convergence of measures in separable Banch space : *Chanrasia, A. R.*

The aim of this paper is to obtain the necessary and sufficient conditions for relative compactness far a new type of convergence which has applications in the field of probabillty theory.

104. A characterization of strongly measurable Pettis fntegrable functions : *G. Rajam (Madurai).*

In this paper, we characterize the strongly measurable Pettis integrable functions as those functions which can be expressed as a series $\Sigma x_n K_{A_n}$ a. e. with $\Sigma x_n\ \mu(An)$ converging unconditionally. This enables us to deduce that a strongly measurable function is Pettis integrable if it is weakly integrable on the whole space. Then we collect several equivalent characterizations of strongly measurable Pettis integrable functions. Let (S, Σ, μ) be a σ finite measure space, X a Banach space and let $f : S \rightarrow X$. The following are equivalent : (i) f is D-1 integrable, (ii) f is

strongly measurable and Pettis integrable (*iii*) f is strongly measurable and $f=g+h$ a. e. where g is a bounded Bochner integrable function and h is a countable valued function. Also if $h=\Sigma x_i K_{A_i}$ pairwise disjoint, then $\Sigma x_i \mu (A_i \cap A)$ converges unconditionally for each $A \in \Sigma$ (*iv*) $f=\Sigma x_i K_{A_i}$ a. e. where the series converges absolutely a. e. and $\Sigma x_i \mu (A_i \cap A)$ converges unconditionally for each $A \in \Sigma$ (*v*) $f=\Sigma x_i K_{A_i}$ a. e. and $\Sigma x_i \mu (A_i)$ converges unconditionally.

105. Convolution product of B-valued almost periodic functions : *K. Venkataraman.*

Let $F(G)$ denote the set of all functions from a group G into a fixed complete atomic Boolean algebra $B=2^x$ endowed with the cartesian products (B, d, M) topology T on B defined by the B-metric $d(a, b)=a \oplus b$ and the dual ideal M consisting of complements of finite sums of atoms of B and 1. Let $A(G)$ denote the set of all B- valued almostperiodic functions with relative topology as a subspace of $F(G)$. Then we prove

(1) Let $f, g \in A(G)$. Then the convolution product $f \times g(x)$ is B- valued almostperiodic and $A(G)$ is a B-Normed B-algebra with respect convolution product (2) Given $f \in A(G)$, $p \in M$, $f(x) \oplus f \times X_{H_p}(x) \leqslant p$ where X_{H_p} is the characteristic function of the p-Kernel H_p of f. (3) The almostperiodic functions on G can be uniformly approximated by finite symmetric differences of the left (or right) translates of the characteristic functions of the normal subgroups of finite index in G.

106. On error bounds in strong approximations for the eigenvalue problems : *R.P. Kulkarni and B.V. Limaye (Bombay).*

Some corrections of error bounds obtained by Chatelin and Lemordant for the first three terms of the asymptotic series expansion of the eigenvalues and spectral projections in the case of a strong approximation are given. The error bounds for the approximations of order 2 in Galerkin's method are compared with the Rayleigh quotients constructed with the eigenvectors in Sloan's method. Some numerical experiments are also carried out.

107. On expectations in locally convex *-Algebras : *R. N. Mukherjee (Varanasi).*

In the present paper we extend the notion of Expectation functionals (as introduced by D. Kannan in an earlier work) on locally convex *-algebras. We study some important and interesting properties of these functionals. We also describe some simple relationship which exists

between quasi-expectation functionals which are representable and quasi-expectation operators of Nakamura and Turumaru on a C^*-algebra.

108. Bipositive isomorphisms of multiplier algebras of Segal algebras : *Kasturi Ramanath (Madurai).*

Let G_1 and G_2 be two locally compact abelian groups. Segal algebras are dense translation invariant subalgebras of the group algebras which are Banach algebras in their own right. Let $S_1(G_1)$ and $S_2(G_2)$ be two Segal algebras on G_1 and G_2 respectively. If $M(S_1(G_1))$ and $M(S_2(G_2))$ are the multiplier algebras of $S_1(G_1)$ and $S_2(G_2)$, then it is proved in this paper that a bipositive isomorphism of $M(S_1(G_1))$ into $M(S_2(G_2))$ induces a topological isomorphism of the underlying groups G_1 and G_2.

109. Rdmarks on a space of Zygmnnd : *Kasturi Ramanath (Madurai).*

Let $0<\alpha<2\phi$. Let $\wedge_\alpha$ denote the class of all continuous complex valued functions f on the real line R with period l such that there exists a constant K satisfying the condition

$$\sup_{x\in R} | f(x+t)-2f(x)+f(x-t) | \leqslant k | t |^\alpha \text{ as } t\to 0,$$

Then Unni and Keshavamoorthy have proved that $\wedge_\alpha$ is a Banach space with norm

$$\|f\| = \sup_{\rho,x,t\in R} \left\{ | f(\rho) |, \frac{| f(x+t)+f(x-t)-2f(x) |}{| t |^\alpha} \right. ;$$

Also λ_α is a closed linear subspace of $\wedge_\alpha$ where $\lambda_\alpha = \{f :$

$$\left. \sup_{x\in R} \frac{| f(x+t)-2f(x)+f(x-t) |}{| t |^\alpha} \to 0 \text{ as } t\to 0 \right\}.$$

In this paper we have the following results

Theorem 1. If T is linear isometry of λ_α into itself, there exists a complex number λ and a real ρ such that either

$$Tf(x) = \lambda f(\rho - x),\ x\in R \quad \text{or}$$
$$Tf(x) = \lambda f(x+\rho) \text{ and } | \lambda | = 1.$$

Theorem 2. f is a multiplier from L^∞ into $\wedge_\alpha$ iff $f\in \wedge_\alpha^1$ where $\wedge_\alpha^1$ is the set of all periodic functions of period 1 on the real line, integrable on [0, 1] satisfying

$$\int_0^1 | f(x+t)+f(x-t)-2f(x) | \, dx \leqslant B \, | t |^{\alpha}$$

for some constant B for all $t \in R$.

Theorem 3. f is a multiplier from L into Λ_α iff $f \in \Lambda_\alpha$ and f is a multiplier from L^1 into λ_α iff $f \in \lambda_\alpha$.

110. A^p (G) is not a dual space : *Kasturi Ramanath* (*Madurai*).

Let G be a locally compact abelian group with character group Γ. For $1 \leqslant p < \infty$, let $L^p(G)$ be the equivalence classes of functions whose pth powers are absolutely integrable with respect to the Haar measure on G. Let $M_{bd}(G)$ be the set of all bounded Radon measures on G. Let $A^p(G) = \{f \in L^1(G) : \hat{f} \in L^p(\Gamma)\}$, with norm $\| f \|^p = \| f \|_1 + \| \hat{f} \|_p$. Then Larson has raised the question as to whether $A^p(G)$ is actually a dual space for $p > 2$. Using the following characterisation of the multipliers on $A^p(G)$ the question is answered in the negative.

Theorem 1. If $p > 2$ the space of multipliers from $L^1(G)$ to $A^p(G)$ is isometrically isomorphic to $B^p(G)$ where

$$B^p(G) = \{\mu \in M_{bd}(G) : \hat{\mu} \in L^p(\Gamma)\}$$

the norm on $B(G)$ being given by

$$\| \mu \|^p = \| \mu \| + \| \hat{\mu} \|_p.$$

Theorem 2. For $p > 2$, $A^p(G)$ is not a dual space.

111. Altman's contractors and matkowskis fixed point theorem : *P. V. Subrahmanyam* (*Hyderabad*) *and K. Balakrishna Reddy* (*Madras*).

Matkowski (Integrable solutions of functional equations, Dissertationes Math. CXXVII (1975), 1—68) proved a fixed point theorem generalizing Banach's contraction principle and this theorem has found applications in the solution of various systems of functional equations. Altman (Contractors and Contractor directions ; Theory and applications ; Marcel Dekker, 1977) developed his theory of contractors and contractor directions for the study of nonlinear operator equations. He was led to the concepts of contractors and contractor directions by the fact that in "many applied problems leading to operator equations the inverse operator is required to exist and to be even continuous". Altman's theory

of contractors offers not only existence and convergence theorems but also unifies in a significant manner a large class of iterative methods such as the method of successive approximations, the Newton-Kantorovitch method and the method of steepest descent. These apart, his theory yields the contraction principle and Krasnoselskii's fixed point theorem as corollaries.

This paper in five sections provides a common basis for the essential ideas of Altman and Matkowski in solving operator equations. The concepts and results basic to this paper form § 1. The main theorem of § 2 unifies the apparently disconnected theorems of Matkowski and Altman. The coincidence theorem in § 3 extends further Altman's generalization of Krasnoselskii's fixed point theorem. § 4 deals briefly with situations featuring nonlinear majorants. The concluding § 5 gives an application of our theory for the solution of a system of evolution equations in Banach spaces, besides an implicit function theorem.

112. Some fixed point theorem for mappings satisfying rational expressions: *R. K. Bose and M. K. Rathi (Pilani).*

Following the technique first applied by Harinath (Ind. J. of Pure and Appl. Math., 1484—90, Dec., 1979) we have extended some recently proved results to pseudo-compact Tichonov spaces. We have generalized the results of Phulendu Das (Ind. J. of Pure and Appl. Math., 11 (2), Feb., 1980) and that of Haimabati Chatterji (1979). A common fixed point theorem and a fixed point theorem concerning a condensing mapping are also obtained. Three theorems are stated below :

Theorem 1. Let X be a pseudo-compact Tichonov space and let F denote a non-negative real-valued continuous function over $X \times X$. If $T : X \to X$ is a continuous map satisfying

$$F(Tx, Ty) < \frac{aF(Tx, x)\, F(Ty, y)}{F(x, y)} + bF(x, y)$$

for distinct $x, y \in X$, where $a+b \leqslant 1$, $a<1$, $F(x, y)=x=y$, then T has a fixed point. Further it is unique if $b \leqslant 1$.

Theorem 2. Let X and F be as given in Theorem 1. Let $T : X \to X$ be a continuous mapping satisfying the following :

$$F(Tx, Ty) < \frac{aF(Ty, y)\,[1+F(Tx, x)]}{1+F(x, y)} + bF(x, y)$$

where $a+b \leqslant 1$, $a<1$. Then T has a fixed point in X which is unique whenever $b \leqslant 1$.

Theorem 3. Let (X, d) be a complete metric space and let F be a non-negative real-valued function $X \times X$ which is continuous and symmetric. Let $T : X \to X$ be condensing such that :

$$F(Tx, Ty) < \frac{aF(y, Ty)\,[1+F(x, Tx)]}{1+F(x, y)} + bF(x, y)$$

$$x, y \in X,\ a+b \leqslant 1,\ a,\ b > 0.$$

If for some $x_0 \in X$, the sequence $\{T^n x_0\}$ is bounded, then T has a unique fixed point.

113. The metric projection bound and Lipschitz constant for the radial projection in normed spaces : *O.P. Kapoor, S.B. Mathur (Kanpur).*

Let P_M denote the set-valued metric projection on the proximinal subspace M of a real normed linear space X. The norm of P_M is defined by

$$\| P_M \| = \text{Sup}\,\{ \| z \| : z \in P_M(x) \text{ and } \| x \| \leqslant 1 \}$$

and the metric projection bound of X is defined by

$$MPB(X) = \text{Sup}\,\{ \| P_M \| : M \text{ proximinal subspace of } X \}.$$

It is known that $1 \leqslant MPB(X) \leqslant 2$, $MPB(X)=1$ if X is a Hilbert space, and that $MPB(X)=2$ if and only if the space X is not uniformly nonsquare. The mapping $T : X \to X$ defined by

$$T(x) = x, \text{ if } \| x \| \leqslant 1$$

$$= \frac{x}{\| x \|}, \text{ if } \| x \| \geqslant 1,$$

is called the radial projection onto the unit ball of X. It is well known that T is Lipschitz and the Lipschitz constant $k(X)$ lies between 1 and 2. In this note we show that the metric projection bound and the Lipschitz-constant are equal and thus exhibit the connection between the results of R. L. Thele [Proc. Amer. Math. Soc. 42 (1974) 483—486] and others about $k(X)$ on the one hand and those of M. A. Smith, F. Deutsch and J. Lambert about $MPB(X)$ on the other. We also obtain a formula for the Lipschitz constant in the l_p spaces and compute for example

$$k(l_3^2) \text{ and } k(l_4^2).$$

V SUMMABILITY, FOURIER ANALYSIS, REAL AND COMPLEX ANALYSIS

114. On the Lebesgue constants for Tayler means and [S, α_n] means : *V. Swaminathan (Trivandrum).*

The Lebesgue constants for $[S, \alpha_n]$ means have been obtained by Meir and Sharma for

$$0 < \alpha_n \leqslant q < 1 \quad (n = 0, 1, \ldots).$$

The relaxation of the condition $\alpha_n \leqslant q$ (where $q < 1$) has been left open in their paper. The object of the present note is to study those cases where such an upper bound for α_n is not possible. The discussion indicates two distinct possibilities, depending on whether V_n^2 / T_n is bounded or unbounded, where

$$V_n = 1 + 2 \sum_{j=0}^{n} \frac{\alpha_j}{1 - \alpha_j} \quad \text{and} \quad T_n = 2 \sum_{j=0}^{n} \frac{\alpha_j}{(1 - \alpha_j)^2}$$

The corresponding results for the Taylor means due to Powell follow on analogous lines.

115. A note on the [K, μ_n] family of summability methods : *V. Swaminathan (Trivandrum).*

The $(K, \mu_n]$ family of summability methods was introduced by Jakimovski. It was shown that this method is regular if and only if

$$\mu_n = \int_0^1 t^{n+1} \, d\alpha(t) \quad (\text{for } n \geqslant 0)$$

where $\alpha(t)$ is a function of bounded variation in the closed interval $[0, 1]$ and that $\alpha(1) - \alpha(0) = 1$. In the present paper, we investigate the domain of summability of a series of Legendre polynomials by the $[K, \mu_n]$ method. The corresponding result for the Abel transform, which is already known, follows as a special case of the above.

116. Absolute Nörlund summability factors for Fourier series : *Prem Chandra (Ujjain) and R.R. Gupta (Satna).*

Let f be 2π-periodic and L-integrable over $(-\pi, \pi)$. Without loss of generality we may assume its Fourier series to be

$$f(t) \sim \sum_{n=1}^{\infty} (a_n \cos nt + b_n \sin nt) = \sum_{n=1}^{\infty} A_n(t).$$

We write for $c \geqslant 1$, $1 > \pi \exp(\exp(c))$ and fixed real x:

$$\phi(t) = \tfrac{1}{2}\{f(x+f) + f(x-t)\},\ \phi_1(t) = t^{-1}\int_0^t \phi(u)\,du,$$

$$R(t) = \phi(t) - \phi_1(t) \quad \text{and} \quad R_1(t) = \phi_1(t) - t^{-1}\int_0^t \phi_1(u)\,du.$$

Varshney [Proc. American Math. Soc. 10 (1959) 784-789] obtained $\left| N, \dfrac{1}{n+1} \right|$ summability of $\displaystyle\sum_{n=1}^{\infty} \frac{A_n(x)}{\log(n+1)}$ whenever $\phi(t) \epsilon BV(0, \pi)$.
In this paper we have proved the following theorems :

Theorem I. *Let (p_n) be a non-negative and non-increasing sequence such that*

$$\text{(i)} \quad P_n \sum_{k=n}^{\infty} \frac{1}{kP_k \log(k+2)} = O(1).$$

Then, in order that $\displaystyle\sum_{n=1}^{\infty} \frac{A_n(x)}{\log(n-1)}$ $\varepsilon \mid N, P_n \mid$ *whenever* $R(t)\ (\log \log \frac{1}{t})^c$ $\varepsilon BV(0, \pi)$, *it is necessary and sufficient that* $c > 1$. *Further, the factor* (y_n) *cannot be replaced by* (y_n^{β}) $(0 < \beta < 1)$.

Theorem 2. *Let (p_n) be a non-negative and non-increasing sequence such that (i) holds and* $(n\, p_n \{\log(n+2)\}/p_n)\ \varepsilon\ bv$. *Then, in order that*

$$\sum_{n=1}^{\infty} \frac{A_n(x)}{\log(n+1)} \ \varepsilon \mid (C, 1)(N, p_n) \mid,$$

whenever $R_1(t)(\log \log \frac{1}{t})^c \in BV(0, \pi)$, *it is necessary and sufficient that* $c > 1$. *Further,* (y_n) *cannot be replaced by* (y_n^{β}) $(0 < \beta < 1)$.

117. On the absolute summability of associated Fourier series : *Sarjoo Prasad Yadav (Tikamgarh).*

Let $f(t)$ $(-\pi, \pi)$ and be 2π-periodic function. Fourier series corresponding to the function $f(t)$ at $t = x$ is given by

(1.1) $$\tfrac{1}{2}a_0+\sum_{1}^{\infty}(a_n\cos nx+b_n\sin nx)\equiv\sum_{0}^{\infty}A_n$$

We denote the nth partial sum of (1.1) by S_n and use the following notations :

(1.2) $$\varphi(t)=\tfrac{1}{2}\{f(x+t)+f(x-t)-2s\}$$

(1.3) $$\varphi_1(t)=1/t\int_0^t\varphi(u)du$$

Following theorems are proved :

Theorem 1. *Let $x(v)$ satisfy the following conditions (a) For $v\geqslant 1/\pi$ $x(v)$ is positive and continuous. Also $x(v)$ is bounded in any finite interval in $v\geqslant 1/\pi$. (b) There is a non-negative constant $\lambda<1$ such that for sufficiently large v, $v^{\lambda}x(v)$ is non-decreasing and $v^{-\lambda}$ $x(v)$ is non-increasing. Suppose that*

(i) $x(1/t)\varphi_1(t)\in BV(0,\pi)$

(ii) $\{x(1/t)\varphi_1(t)/t\}\in L(0,\pi)$.

Then the series

$$\sum^{\infty}\frac{(s_n-s)x(n)}{n}$$

is summable $|C,1|$. And if further $x(v)$ is bounded monotonic non-decreasing then the above mentioned series is summable $|C,\alpha|$, $\alpha>\lambda$.

Theorem 2. *If $\varphi_1(t)\in BV(0,\pi)$ and $\varphi_1(t)/\{t\log k/t\}\in L(0,\pi)$ then the series*

$$\sum^{\infty}\frac{s_n-s}{n\log n}\in|C,\alpha|,\ \alpha>0.$$

118. On $|\bar{N},p_n|$ summability of an associated Fourier series : *G.C. Hotta and T.C. Padhi (Ganjam).*

In 1950, R. Mohanty (Jour. Lond. Math. Soc. Vol. 25 part I pp 67-72) proved the following theorem on the Summability $|R,\log\omega,1|$ of an associated Fourier series.

Theorem A : If $\phi(t)/\{\log k/t\}$ $(k>\pi)$ is of bounded variation in $(0,\eta)$, $\eta>0$, then $\Sigma A_n(x)/\log(n+1)$ is summable $|R,\log\omega,1|$, where

$$\varphi(t)=\int_t^{\pi}\frac{\varphi(u)}{u}du.$$

In the present paper we generalise the above theorem by extending it to $|\bar{N},p_n|$ summability. We prove :

Theorem. If $\phi(t)/\log(k/t)$, $(k>\pi)$, is of bounded variation in $(0,\eta)$, $\eta>0$, then the series $\sum_{n=1}^{\infty} \frac{A_n(x)}{\log(n+1)}$

is summable $|\bar{N},p_n|$, where $\{p_n\}$ is a sequence of non-negative decreasing numbers such that the sequence $\{p_n/\log(n+1)\}$ is non-increasing.

119. Some applications of the absolute Euler summability to Legendre series : *Prem Chandra and M.M. Sharma (Ujjain).*

Let $\Sigma\, a_n P_n(x)$ be the Legendre series at x of a Lebesgue-measurable function over $[-1,1]$ and let for $0<\eta<1$, $0<\delta\leqslant\pi$, we write

$$(*) \qquad \int_0^{\delta} \left(\log \frac{k}{|\theta-\phi|} \right)^{\eta} |\, df(\cos\phi)\,| <\infty, \ (k>\pi e)$$

where $\theta=\cos^{-1}x$ for $-1<x<1$. Then we prove the following :

Theorem 1. *If* $\delta<\pi$ *and* $\eta=1$, *then* (*) implies that $\Sigma a_n P_n(x) \in |E,q|$, $(q>0)$ *provided*

$$\int_0^t |f(-\cos\phi)|\, d\phi = O(|h(t^{-1})|) \ (t\to 0+) \ \textit{and}\ \Sigma\, |h(n)|\,(n+1)^{-1/2}<\infty.$$

Theorem 2. *If* $\delta=\pi$ *and* $\eta=1$, *then* (*) *implies* $\Sigma a_n P_n(x) \in |E,q|$ $(q>0)$. *However* (*) *with* $\delta=\pi$ *and* $0<\eta<1$ *is not a sufficient condition.*

Theorem 3. $f(t) \in BV$ $[-1,1]$ *implies* $\Sigma a_n P_n(x)/\log(n+2) \in |E,q|$ $(q>0)$.

120. Absolute Riesz summability of series associated with Legendre series : *Prem Chandra and M.M. Sharma (Ujjain).*

Let $\Sigma a_n P_n(x)$ be the Legendre series at a point x of a Lebesgue-measurable function f over $[-1,1]$. Then in this paper we prove the following main theorem, concerning absolute Riesz summability factors of Legendre series, which yields some interesting new results :

Theorem. *Let* $h(t)$ *be non-negative and non-decreasing with* $t\geqslant 0$ *and let*

$$\int_t^{\pi} \phi^{-1} \; h(\phi)\, d\phi = O(\mu(1/t)) \;\; (t \to 0+),$$

where μ is positive and non-decreasing function.
Then

$$\int_0^t | \; f(\pm \cos u) \; | \, du = O(h(t)\sqrt{t}) \;\; (t \to 0+)$$

and

$$\int_0^t | \, f(\cos(\theta - u)) \, | \, du = O(t) \;\; (t \to 0)$$

imply $\Sigma a_n P_n(x)\, y(n) \in | \, R, \lambda(w), 1 \, |$, *whenever*

(i) $\{\lambda(n) y(n)\}$ *does not decrease for* $n \geqslant n_0$,

(ii) $\Sigma \lambda(n) y(n) n^{-1} = O \left\{ \dfrac{\lambda^2(w)\, y(w)}{w \lambda^{(1)}(w)} \right\}$,

(iii) $\int_l^{\infty} \{\lambda^{(1)}(w)\, y(w) \log w / \lambda(w)\}\, dw < \infty$ *and*

$\int_l^{\infty} \{y(w)\, \lambda(w)/w\}\, dw < \infty$, *where l is some positive number and* $\theta = \cos^{-1} x$ *for* $-1 < x < 1$.

121. On generalized Lototsky summability : *C. Jayasri, (Kariavattom).*

Let $\{h_i(z)\}$ be a sequence of functions analytic on the closed unit disc $U = \{z : | \, z \, | \leqslant 1\}$ and let $\{ f_i(z)\}$ be a sequence of entire functions. For each $z \in U$ define a matrix $a_{nk}(z))$ by the relations

$$a_{00}(z) = 1 \; ; \; a_{0k}(z) = 0 \;\; (k > 0)$$

$$\prod_{i=1}^{n} \; (f_i(w) h_i(z) + 1 - h_i(z) - \lambda \; = \; \sum_{k=0}^{\infty} a_{nk}(z) w^k. \tag{1.1}$$

Denote by $L(f_i, h_i, z)$ the summability transform associated with the matrix $(a_{nk}(z))$. When $h_i = \dfrac{1}{1+d_i}$ is a bounded sequence of scalars, and $f_i(z) = f(z)$ for each i and $f_i(1) = 1$, then (a_{nk}) represents the (f, d_n) matrix. With the further assumption that $f(z) = z$ we have (F, d_n) matrix. Both the (f, d_n) and (F, d_n) matrices are generalizations of the Lototsky matrix. In this paper we study the summability properties of the $L(f_i, h_i, z)$ transform.

122. Quaternionic analysis : *M. Nagaraj and K.E. Bakkesa Swamy.*

R. Fueter (1935) gave the definitions of left and right regular quaternionic functions of a quaternion variable by means of an analogue of the Cauchy Riemann equations. In this paper we have proved that a mapping f: $\mathbf{H} \rightarrow \mathbf{H}$ is conformal if and only if the components f^i $(i=0, 1,2,3)$ of f are mutually orthogonal and $| \Box f^i | = | \Box f^j |$ $(i,j=0,1,2,3)$ where $\Box$ is the quaternian gradient operator. We have also shown that the most general conformal transformation of $\mathbf{H}$ to $\mathbf{H}$ is of the type $(aq+c)(dq+f)^{-1}$. Furthere we have shown that the conformal transformation of $\mathbf{H}$ to $\mathbf{H}$ is a natural generalization of left (or right) differentiable functions.

123. On the iteration of polynomials of degree 4 with real coefficients : *P. Bhattacharyya, Y. Eben Arumaraj.*

Let f be a function of the complex variable z and f_n be the sequence of iterates defined by $f_0\ (z)=z, f_{n+1}(z)=f(f_n(z))$, $n=0,1,2,\ldots$

The Fatou set $F(f)$ is the set of all points z in no neighbourhood of which the sequence $\{f_n(z)\}$ form a normal family in the sense of *Montel*. If f is a rational function of order atleast 2 or a transcendental entire function then $F(f)$ is a nonempty perfect set. To decide the structure of $F(f)$ for a given f is a problem of considerable difficulty.

The structure of the Fatou set $F(f)$ where f is a polynomial of degree 4 is investigated in detail in this paper. It is shown that the structure of the set $F(f)$ depends on the coefficients of the polynomial in a very complicated way.

124. On the existence of iterative roots of polynomials of degree 3 : *P. Bhattacharyya, Indra Selvaraj.*

Let f be any function of the complex variable z. The natural interates of f are defined inductively by

$$f_0(z)=z,\ f_n(z)=f(f_{n-1}(z)), \qquad n=1,2,\ldots.$$

If $f_r=g$, we say that f is an *iterative root* of g of order r.

In this paper we prove

Theorem. *Let* $P(z)=az^3+bz+c$. *Then* $P(z)$ *has no iterative root of order* 2.

When $P(z)=az^2+bz+c$ it is already known that $P(z)$ has no iterative root of any order.

We conjecture that if $P(z)$ is a polynomial of degree n, where n is a prime number, then it cannot be the second iterate of any function $f(z)$ whatsover. By 'whatsover' we mean all functions and not only continuous or differentiable functions considered in analysis.

125. **Algebraic structure of discrete pseudoanalytic functions :** *Smt. Mercy K. Jacob (Cochin).*

In this paper algebraic structure of pseudo q-analytic functions is discussed. Defining a convolution it is shown that a subclass of the class of S-pseudoqanalytic functions forms a commutative ring with unit element under the operations $+$ and (*) where $+$ is the ordinary addition and (*) is the convolution.

126. **On the set of normality in iteration theory :** *P. Bhattacharyya.*

Let f be a nonconstant function of the complex variable z, which is either transcendental or has two isolated essential singularities. Let $C(f)$ denote the set of normality (in the sense of Montel) of the iterates $\{f_n(z)\}$ of f.

We prove

Theorem 1. *The number of completely invariant components of $C(f)$ is atmost one.*

A set S is said to be completely invariant under $z \to f(z)$ if $\alpha \in S$ implies $f(\alpha) \varepsilon S$ and if $\beta \varepsilon S$ for every solution β of $f(\beta)=\alpha$.

Theorem 2. *The number of disjoint components of $C(f)$ is 0,1 or ∞.*

Theorem 3. *Let $C(f)$ have infinite number of components of which G is completely invariant. Then there does not exist any component G_1 of $C(f)$ which is isolated from all the components of $C(f)$ other than G.*

Theorem 4. *$C(f)$ has exactly one component for $f=e^z-1$.*

127. **On a sub-family of univalent functions :** *N.K. Thakare (Aurangabad) and S.R. Kulkarni (Sangli).*

A new subfamily, denoted by $D(\alpha,\beta,\zeta)$, of the class of holomorphic, normalized univalent functions in the unit disc in the complex plane satisfying the condition

$$\left| \frac{f^1(z)-1}{2\zeta(f^1(z)-\alpha)-(f^1(z)-1)} \right| < \beta$$

where $\beta \in (0, 1]$, $1/2 \leqslant \zeta \leqslant 1$, $0 \leqslant \alpha < 1/2\zeta$ is introduced.

A characterization in terms of integral representation of members of $D(\alpha,\beta,\zeta)$ is accomplished. Sharp bounds on the sizes of $|f|$, $|f'|$, and $\text{Re}(f')$ where $f \in D(\alpha,\beta,\zeta)$ are obtained. Sharp estimates on the arc length of the image of $|z|=r$, $(0<r<1)$, under members of $D(\alpha,\beta,\zeta)$ and on the area of the image of $|z|<r$, $(0<r<1)$ are given.

The considerations have been specialized to those members of $D(\alpha,\beta,\zeta)$ which have negative coefficients.

128. On univalent holomorphic functions : *N.K. Thakare (Aurangabad) and S.R. Kulkarni (Sangli).*

Let $S(\alpha,\beta,\zeta)$ denote the subfamily of the family of normalized and univalent functions in the unit disc of the complex plane satisfying the condition.

$$\left| \frac{zf'/f-1}{2\zeta(zf'/f-\alpha)-(zf'/f-1)} \right| < \beta$$

where $\beta \in (0,1]$, $\frac{1}{2} \leqslant \zeta \leqslant 1$ and $0 \leqslant \alpha < 1/2\zeta$.

Members of $S(\alpha,\beta,\zeta)$ have been characterized. The holomorphic functions with negative coefficients and that are in $S(\alpha,\beta,\zeta)$ are also characterized and several properties are obtained. The radii of close to convexity of several integrals involving members of $S(\alpha,\beta,\zeta)$ and polynomials of degree n whose all the zeros lie outside or on the unit circle are determined. Finally, we also determine the span of the index parameter δ in the integrals of the form

$$\int_0^z (f(t)/t)^{\delta} (p(t))^{l/n} \, dt$$

where $f \in S(\alpha,\beta,\zeta)$ and p is a polynomial of degree n whose all the zeros lie outside or on the unit circle.

129. A remark on the arithmetic mean of convex univalent functions : *V. Karunakaran, (Madurai).*

In this paper we settle the problem of determining the best possible value for R such that $2^{-1}(f+g)$ maps $|z|<R$ onto a convex domain whenever f and g are normalised convex univalent functions in the unit disc of the complex plane. Labelle and Rahman showed that R can be taken as the smallest positive root of the equation $1-3x+2x^2-2x^3=0$ but this result is not sharp. We show that the best possible value for R is $(\sqrt{2}-1)$.

130. Non Euclidean distortions for linear contracting families of locally univalent analytic functions : *G.P. Kapoor and A.K. Mishra (IIT, Kanpur).*

Let L.U. denote the family of locally univalent analytic fuctnions in

$D=\{z: |z| <1\}$ and let U denote the family of univalent functions in L.U. For any family $Y \subset$ L.U., let Y^* denote the family of functions $F(z)=(f(z)-f(0))/f'(0)$, for f in Y. A family $Y \subset L.U.$ is called linear contracting if for every f in Y and every conformal self mapping ϕ of D, the function $\omega_\phi[f(z)] =[f(\phi(z))-f(\phi(0))]/\phi'(0)f'(\phi(0))$ is in Y^*. The order of a linear contracting family Y is defined as $\alpha=\sup\{ |\omega'(0)| /2: \omega \in Y^*\}$. The distortion theorems for the linear contracting family U^* are well known. Recently, Ky Fan (J. Math. Anal. Appl. 66(3), (1978), 626-631), has obtained distortions for the linear contracting family U in terms of the non Euclidean metric. In the present paper, the results of Ky Fan are extended for any linear contracting family. A sample result follows : Let Y be a linear contracting family of order α and $f \in Y$. Let $F=f(D)$. For $w_1, w_2 \in F$, define $d(w_1, w_2)= \inf_V (\text{diam } V)$, where V is any continuous curve in F joining w_1 and w_2. Then, for $|z|=r<1$,

$$\frac{1}{2\alpha}[1-e^{-2\alpha p(u,v)}] \leqslant d\left(\frac{f(u)-f(v)}{(1-|v|^2)f'(v)}, 0\right) \leqslant \frac{1}{2\alpha}[e^{2\alpha p(u,v)}-1]$$

where $p(u,v)$ is the non Euclidean distance between arbitrary points u and v in D.

131. **On some classes of regular univalent functions :** *S.L. Shukla and G.P. Bhargava.*

Let $S(m,M)$ be the class of regular functions $f(z)=z+\ldots$ for which $\left| \frac{zf'(z)}{f(z)} - m \right| < M$, $|m-1| < M \leqslant m$ in $|z|<1$. We show that integral operators of the form

$$F(z)=\left[\frac{c+p}{z^c}\int_0^z t^{c-1} f(t)^p dt\right]^{1/p}=z+\ldots$$

for suitable choices of the parameters c and p transforms $S(m,M)$ into $S(m,M)$. Our result generalizes recent results (Theorem 2) [S. Miller, P. Mocanu and M. Reade, Starlike Integral Operators, Pacific J. Math., to appear]. With other restrictions on the parameters, we obtain transformations of the set $S^*(\alpha) \times S(m,M)$ into $S^*(\alpha)$, $S^*(\alpha)$ denotes, as usual, the set of all regular (normalized) univalent functions starlike of order α in $|z| \leqslant 1$, $0 \leqslant \alpha < 1$.

131 a. **The radius univalence of certain regular functions :** *G.P. Bhargawa (Kanpur) and S.P. Dwivedi (Kharakvasla).*

Let $P(\alpha)$ be the class of functions p given by $p(z)=1+c_1z+\ldots$ regular in $D=\{z: |z| <1\}$ and satisfying the condition Re $(p(z)) \geqslant \alpha$, $z \in D$, $0 \leqslant \alpha \leqslant 1$. Let $S(m,M)$ be the class of functions f given by

$f(z)=z+\ldots$

regular in D and satisfying the condition

$$\left| \frac{zf'(z)}{f(z)} - m \right| < M \text{ for } z \in D,\ (m,M) \in E$$

where $E = \{(m,M) : |m-1| < M \leqslant m\}$.

In this paper we obtain the radius of univalence and starlikeness of the set of all functions f that are regular in D and are defined by $f(z) = s(z)p(z)q(z)$ where $s \in S(m,S)$, $p \in P(\alpha)$ or $1/p \in P(\alpha)$ and $q \in P(\beta)$ or $1/q \in P(\beta)$. These results are sharp and include the results of Ziegler [Math. Z., 122 (1971), 351-54], Causey and Merkes [J. Math. Anal. Appl., 31 (1970), 579-86] and Ratti [Math. Z., 107 (1968), 241-48].

132. **On distortion and radius of starlikeness of schlicht functions :** *V.P. Gupta and Iqbal Ahmad,*

Let $P_n(\alpha)$ denote the class of analytic functions $p(z) = 1 + c_n z^n + \dots$ satisfying in the unit disc E the condition $\text{Re}\{p(z)\} > \alpha$. $0 \leqslant \alpha < 1$. The class $P_n^\rho(\alpha)$ consists of functions $p \in P_n(\alpha)$ having $|c_n| = \rho$. Let $C_n^\rho(\alpha)$, $S_n^\rho(\alpha)$ denote the classes of functions $f(z) = z + a_{n+1}z^{n+1} + \dots$ which are respectively convex and starlike functions of order α with second coefficient fixed. Let $P_n(\alpha, \beta)$ and $D_n(\alpha,\beta)$ denote the classes of analytic functions f which satisfy in E the condition

$$\text{Re}\left\{\frac{f(z)}{g(z)}\right\} > \beta,\ 0 \leqslant \beta < 1 \text{ for } \frac{g}{z} \in P_n^\rho(\alpha) \text{ and } g \in S_n^\rho(\alpha)$$

respectively.

In this paper distortion properties of functions belonging to these classes along with the radius of univalence and starlikeness of functions belonging to $D_n(\alpha,\beta)$ are obtained.

133. **Extensions of Ritt growth concepts and same fundamental growth theorems connected with entire Dirichlet series in C^k :** *J. Gopala Krishna, I.H.N. Rao and K.H.S. Subrahmanyam (Waltair).*

In 1977, Gapala Krishna and Nagaraja Rao (J. Ind. Math. Soc. Vol. 41 (1977) pp 203-219) considered the maximum term μ and the maximum modulus M of an entire power series in C^k and established that $\log M$ and $\log \mu$ have the same growth *w.r.t.* Borel and Lindelof families. The main interest of this work is to extend these results to the case of entire Dirichlet series in C^k and to establish that $\log M$ and $\log \mu$ have the same growth *w.r.t.* various Ritt families without any restrictions on the Ritt orders. In

particular the Probabilistic techniques are used to prove that $\log M(x) \sim \log \mu(x)$ as $x \to +\infty$ when the Dirichlet series (in C^k) is of finite Ritt order.

134. On the type of analytic Dirichlet series of fast growth : *Krishna Nandan and RSL Srivastava (Kanpur).*

Let $\sum_{n=1}^{\infty} a_n \exp(s\lambda_n)$, where $0 \leqslant \lambda_1 < \lambda_2 \ldots < \lambda_n \to \infty, s = \sigma + it$ (σ and t, real. variables) subject to $\limsup_{n\to\infty} n/\lambda_n = D < \infty$ represent a holomonphic function $f(s)$ in the half-plane $\sigma < A (-\infty < A < \infty)$, where $A = -\lim\sup (\log |a_n|)/\lambda_n$. The function $f(s)$ is said to be of order ρ if $\rho = \limsup_{\sigma\to A} \log \log M(\sigma)/[-\log\ (1-\exp(\sigma - A))]$. If $f(s)$ is of fast growth the function $f(s)$ is said to be of H_p-order $\rho(p)$ and of lower H_p-order $\lambda(p)$ if and only if

$$\lim_{\sigma\to A} \begin{matrix}\sup \\ \inf\end{matrix}\ l_p M(\sigma)/[-\log\{1-\exp\ (\sigma - A)\}] = \begin{matrix}\rho(p), \\ \lambda(p)\end{matrix}$$

where we use the familiar abbreviation $l_p x = \log\log\ldots$ (p times) $x (p=1,2\ldots)$ observing that $l_p x > 0$ for real x after a stage. If order is non-zero finite its type concept is defined as

$$\lim_{\sigma\to A} \begin{matrix}\sup \\ \inf\end{matrix}\ \frac{l_p M(\sigma)}{\{1-\exp\ (\sigma - A)\}^{-\rho}} = \begin{matrix}T \\ \tau\end{matrix},$$

In the present paper, a coefficient characterisation for the type and lower type of the function of fast growth and a decomposition theorem involving type and lower type are obtained.

135. On the mean values of an entire function and its derivatives represented by Dirichlet series-II : *Ram Prakash Doherey and R. S. L. Srivastava, (I.I.T., Kanpur).*

Mean values ($I_\delta(\sigma, f) = \lim_{T\to\infty} \frac{1}{2t} \int_{-T}^{T} |f(\sigma + i\,t)|^\delta\, dt$

and $I_{\delta,k}(\sigma, f) = e^{-k\sigma} \int_0^\sigma I_\delta(x, f)\, e^{kx}\, dx, 0 < k, \delta < \infty$) of an entire

function, $f(s) = \sum_{k=1}^{\infty} a_k\, e^{s\lambda_k}$, where $\lambda_1 \geqslant 0, \lambda_{k+1} > \lambda_k$, $\lim_{k\to\infty} \lambda_k = \infty$

and a_k is the sequence of complex numbers, and its derivatives have been

studied in the context of Ritt order and type by several workers. For functions of zero order (slow growth) and those of infinite order (fast growth) one needs to introduce new growth indices to study the mean values. The concepts of such types of indices for $f(s)$ were introduced by Krishna Nandan (Some aspects of the growth of analytic functions represented by Dirichlet series, Ph.D. Thesis, Kanpur University, Kanpur 1973). In our first part, we have studied the growth properties of $I_\delta\ (\sigma, f)$ and $I_\delta\ (\sigma, f^{(m)})$, $f^{(m)}$ being the mth derivative of f, corresponding to these indices and in the present paper, we have investigated some growth results for $I_{\delta,k}\ (\sigma, f)$ and $I_{\delta,k}$ $(\sigma, f^{(m)})$ corresponding to the same constants introduced by Krishna-Nandan. These results are equaliy applicable to the functions of slow as well as fast growth. The results thus obtained generalize several known results.

136. On the λ^*—logarithmic type of analytic functions represented by Dirichlet series : *K.N. Awasthi and K.K. Dixit.*

Consider the Dirichlet series

$$(1.1) \qquad f(s) = \sum_{n=1}^{\infty} a_n \exp(s\lambda_n)$$

where $\lambda_{n+1} > \lambda_n,\ \lambda_1 \geqslant 0,\ \lim_{n\to\infty} \lambda_n = \infty,\ s=\sigma+it$

(σ, t being real variables) and

$$(1.2) \qquad \limsup_{n\to\infty} \frac{n}{\lambda_n} = D < \infty.$$

Let D^*_α, as shown by us earlier. denote the class of all functions $f(s)$ of the form (1.1) which satisfy (1.2) and are analytic in the half-plane Re $s<\alpha$ $(-\infty<\alpha<\infty)$ and are of order zero. Let $A^*_\alpha \subset D^*_\alpha$, we say that $f(s) \in A^*_\alpha$, if and only if there exists $\epsilon>0$ such that

$$\frac{m(\sigma, f)}{\{1-\exp(\sigma-\alpha)\}^{-\epsilon}} \to \infty, \text{ as } \sigma\to\alpha.$$

In this paper, by using a simple argument we first show that for a function of irregular logarithmic growth the lower logarithmic type is always zero, and therefore for a function $f(s) \in A^*_\alpha$ of irregular logarithmic growth we define λ^*—logarithmic type t_λ^* and then obtain some relations which connect λ^*—logarithmic type with the coefficients and exponents in the Dirichlet series expansion for $f(s)$. We also show through an example that λ^*—logarithmic type t_{λ_*} is not only non-zero in general but $t_{\lambda_*} > T^*$, the logarithmic type of $f(s)$.

VI DIFFERENTIAL EQATIONS, ASTRONOMY, RELATIVITY

137. The convergence of an algorithm for solving the Lyapunov matrix equation; *C. S. K. Chetty and R. K. S. Rathore (Kanpur).*

Recently, Hoskins, Meek and Walton have proposed an algorithm for solving the Lyapunov Matrix Equation $A^T Q+QA=-C$, claiming that the algorithm converges if A is stable. However, by providing counter-examples elsewhere, Rathore and Chetty pointed out that the main convergence result used by Hoskins et al. is not true whether A is stable with a real spectrum or with a complex spectrum and furthermore suggested a modified algorithm whose theoretical convergence is guaranteed, when A is stable with a real spectrum. The purpose of this paper is to present a theoretical proof of the convergence of the modified algorithm.

138. Solutions of Einstein equations by Kerr-Schild transformations in the axially symmetric case: *M. Nagaraj and R.V. Prabhakara (Bangalore).*

In this paper we have determined a solution for Axially Symmetric Static space-time in terms of the harmonic function when the Space-time admits a geodesic null congruence. In this case the congruence is also hyper-surface orthogona, and shear-free. Using this congruence we obtain other solutions of Einstein equation by Kerr-Schild transformations.

139. On existence of solutions of ordinary differential equations in locally convex spaces—I : *S. B. Agase (Waltair).*

Phillips R. S. in his paper "Integration in a convex linear space, Trans. Amer. Math. Soc. Vol. 17 (1940) pp. 114–145" has introduced the concepts of integral and pseudoderivative of a function with values in a locally convex topological vector space (l. c. t. v. s.) F. In this paper we consider the initial value problem (IVP)

$$x'=f\ (t,\ x),\ x\ (o)=z_0 \quad ...(1)$$

where " ′ " denotes the pseudoderivative, $f\ :[0,1]\times E\rightarrow E$. Phillips has proved an existence of a solution (in his sense) of IVP (1), where E is a metrizable l. c. t. v. s., assuming that f has a compact range. We assume that f is bounded and satisfies a compactness type condition and establish an existence of solution of IVP (1). Our theorem improves the result by Phillips. We have further weakened conditions on f and established another existence theorem.

140. A boundary value problem for functional differential equations of delay type : *Mohd. Faheem and M. Rama Mohana Rao,*

This paper is concerned with the existence and uniqueness of solutions of the boundary value problem.

$$x'(t)=A(t)\,x(t)+f(t, x_t),\ t\,\varepsilon\,[a, b]$$

$$Mx_a+Nx_b=\psi,\ \psi\,\varepsilon\, C,$$

where M and N are bounded linear operators on C, the space of continuous functions. Using a shooting argument, the problem is reformulated as an operator equation. A fixed point theorem is developed and applied to the operator equation. Examples are given to illustrate the results.

141. A note on three point boundary value problems for nonlinear differential equations of the third order : *A. Satyanarayana Rao and B. D. C. N. Prasad.*

In this note we consider boundary value problems of the type.

$$y'''=f(x, y, y', y'')$$

with the boundary conditions

$$AY=\{\lambda_{ij}\}_{3\times 9}\ \{\mu_{ij}\}_{9\times 1}\ =0$$

where $$A=\{\lambda_{ij}\}_{3\times 9}$$

$$\{\mu_{ij}\}^T=\{y(\alpha), y'(\alpha), y''(\alpha), \ldots y''(\gamma)\},\ \alpha<\beta<\gamma,$$

and where uniqueness of solutions is shown with the usual conditions on $f(x, y\, y', y'')$. Besides a close study of Green's functions, use is also made of elementary groups and partial differential equations wherever necessary.

142 Variable mesh cubic spline technique for one-dimensional nonlinear partial differential equation : *Babu Lal Lohar (Roorkee).*

When the flow characteristics are continuous, the uniform mesh finite difference methods can be used for solving problems in fluid mechanics. In case. the flow characteristics have discontinuities in the region of computation, the uniform mesh technique may not provide realistic results unless a very fine mesh is used. Normally, a very fine mesh is required in the

neighbourhood of the points of discontinuities, boudary layers etc. because derivatives there are generally too large. In this article, a Variable Mesh Cubic Spline Technique (VMCST) is proposed for solving such problems. In particular, an algorithm is devised for the solution of Burgers' equation :

$$u_t+u\ u_x=(\delta/2)\ u_{xx} \qquad (1)$$

where $u=u(x, t)$ and δ is the coefficient of diffusivity. For the initial condition $(t=1)$:

$$u(x,\ 1)=x/(1+t_0^{-1/2}\ \exp.\ (x^2/2\delta)) \qquad (2)$$

equation (1) has been solved for various values of δ. Handling of centre of shock (point of discontinuity) is illustrated. Computed results are compared with the exact solution given by Lighthill. They are found in good agreement. Diffusion of the shock with time is shown by graphs.

143. On stability of differential inequalities : *S. P. Shettar* (*Khadakvasla*).

In this paper we consider two differential inequalities

$$\left|\frac{dx}{dt}-f\,(t,\ x)\right| \leqslant \epsilon_1 \qquad (1)$$

$$\left|\frac{dy}{dt}-g\,(t,\ y)\right| \leqslant \epsilon_2 \qquad (2)$$

and define the concept of finite time stability of (1) w. r. to (2). Using a comparison theorem we study that stability properties of these systems depend on the behaviour of solution of a scalar differential equation. For $f\equiv g$ and for a particular kind of scalar differential equation, we get the well known results of the ordinary differential equations. And for $\in_1=0$, $\in_2=0$ our definitions give more general form of finite time stability definitions given by Leonard Weiss and I.F. Infante. We also consider the perturbations given for (1), (2) whenever $\in_1=o$, $\in_2=o$.

VI SPECIAL FUNCTIONS, TRANSFORMS, FUNCTIONAL EQUATIONS, APPROXIMATION THEORY.

144. On the unified presentation of certain classical polynomials : *B. D. Agrawal and Jagadamba Prasad* (*Varanasi*).

This paper attempts to present a unified treatment of the classical orthogonal polynomials, viz. Jacobi, Laguerre, Hermite, Gegenbauer,

Legendre, Tchebichef and their generalizations introduced from time to time. The results obtained in this paper include, explicit form, linear generating relation, operational formula and some other results, for the polynomial set R_n [x; a, b, c, d, p, q, γ, ξ ; w (x)/$n=o$. 1, 2...].

145. Bilateral and trilateral generating relations for a unified polynomial set R_n (x) : *B. D. Agrawal and Jagadamba Prasad (Varanasi).*

The authors have earlier obtained a bilateral generating relation for the unified polynomial set R_n (x). In this paper, we present a more general bilateral generating relation by specializing known results due to Srivastava and Lavoie (1975). We then extend this bilateral generating relation in the form of a mixed trilateral generating relation.

146. Simultaneous saturation theorem for Bernstein polynomials : *R.K.S. Rathore and P.N. Agrawal (Kanpur).*

In the authors' paper (Inverse and Saturation Theorems for Derivatives of Exponential type operators; communicated) the direct and inverse theorems have been obtained in the simultaneous approximation (approximation of derivatives of functions by the derivatives of operators) for the linear combinations of general exponential type operators introduced by May (Saturation and inverse theorems for combinations of a class of exponential-type operators, Canad. J. Math. 28 (1976), 1224–1250) and saturation theorem for regular exponential type operators. So far it is not clear how the saturation theorems could be obtained for general non-regular exponential type operators ; although from the existence of the asymptotic formulae one is led to conjecture that similar saturation theorems hold even in the case of non-regular exponential type operators. May in his paper cited above verified this assertion in the ordinary approximation for the linear combinations of the summation type operators e.g. Bernstein polynomials, Szász operators and Baskakov operators. We, in this paper, extend his study by obtaining the saturation theorem in the simultaneous approximation for the combinations of Bernstein polynomials.

147. Generating functions of the generalised Laguerre polynomials : *G. S. Prasad, (Ranchi).*

New generating functions (bilinear and bilateral) of the generalised Laguerre polynomials $L_n^{\alpha,\beta,\gamma}$ (x) (a new generalisation of Laguerre polynomials L_n (x)) defined by

$$L_n^{\alpha,\beta,\gamma}(x)=\frac{(1+\alpha)_n}{n!}\,{}_2F_2(-n,1+\beta;1+\alpha,1+\gamma;x),$$

n being any non-negative integer, have been obtained. These generating functions yield known generating functions of $L_n^{\alpha}(x)$ and $L_n(x)$ in particular cases. Certain results required in the derivation of the generating functions have also been proved.

148. Dynamical symmetry algebra of ${}_1F_1$ and modified Laguerre polynomials : *Renu Jain and B.M. Agrawal.*

Present paper employs the dynamical symmetry algebra of the confluent hypergeometric function ${}_1F_1$ to derive certain identities which in their turn lead to generating functions for modified Laguerre polynomials which are defined in terms of Laguerre polynomials.

The modified Leguerre polynomials $f_n(x)$ are defined in terms of Laguerre polynomials $L_n^{\alpha}(x)$ by the equation

$$f_n^{\beta}(x)=\frac{(-1)^n}{n!}L_n^{\beta-n}(x)$$

Aforementioned identities directly follow from local multiplier representations of Lie group elements obtained by exponentiating generators of the dynamical symmetry algebra of ${}_1F_1$.

149. Products of several generalized Laguerre polynomials : *J.P. Singhal and Savita Kumari (Baroda).*

For the products of several generalized Laguerre polynomials introduced by us (Jnanabha, 9/10, 1980, to appear) two expansion formulae have been obtained here which provide extension of a result due to Carlitz (Monatsh. Math. 66 (1962). 393-396.)

150 A bilateral generating function for generalised Hermite polynomial : *M. Bhattacharjya.*

Recently we have introduced a new generalisation of the well known Hermite polynomial given by

$$H_n(x;\lambda,r)=x^n{}_2F_0\left(-\frac{n-r}{2},-\frac{n+\lambda+r-1}{2};-;-\frac{1}{x^2}\right).$$

Here we establish a bilateral generating function fcr this genralised class. We prove the following theorem.

$$\text{If } F(x, t) = \sum_{m=0}^{\infty} a_m H_{2m+r} (x ; \lambda, r) t^m$$

$$\text{then } (1+t)^{-\frac{r}{2}-\frac{\lambda+1}{2}} \exp \frac{tx^2}{1+t} F\left(\frac{x}{\sqrt{1+t}}, \frac{ty}{1+t}\right)$$

$$= \sum_{0}^{\infty} b_k(y) \frac{H_{2k+r}(x ; \lambda, r)}{k!} t^k$$

where

$$b_k(y) = \sum_{0}^{k} (-1)^m (-k)_m a_m y^m.$$

We also give an application of the theorem.

151 An inequality for the derivative of a polynomial satisfying $p(z) \equiv z^n$ $p(1/z)$: *Mrs. K. Dewan and N. K. Govil, (New Delhi).*

If $p(z)$ is a polynomial of degree n, then according to a well known result due to S. Bernstein

$$(1.1) \quad \max_{|z|=1} |p'(z)| \leqslant n \max_{|z|=1} |p(z)| .$$

In case $p(z)$ has no zeros in $|z|<1$, it was conjectured by Erdos and proved by Lax [Bull. Amer. Math. Soc. 5ɔ (1944)] that

$$1.2 \quad \max_{|z|=1} |p'(z)| \leqslant \frac{n}{2} \max_{|z|=1} |p(z)| .$$

It was proposed to study the class of polynomials satisfying $p(z) \equiv z^n$ $p(1/z)$ and in this connection it was proved by Govil, Jain and Labelle [Proc. Amer. Math. Soc. 57(1976)] that if $p(z)$ is a polynomial satisfying $p(z) \equiv z^n$ $p(1/z)$ and having all its zeros either in the right half-plane or in the left half-plane, then

$$(1.3) \quad \max_{|z|=1} |p'(z)| \leqslant \frac{n}{\sqrt{2}} \max_{|z|=1} |p(z)|$$

and

$$(1.4) \qquad \max_{|z|=1} |p'(z)| \geqslant \frac{n}{2} \max_{|z|=1} |p(z)| .$$

In this paper we generalize inequality (1.4) of the above result by dropping the condition that $p(z)$ has all its zero either in the right half-plane or in the left half-plane. The result obtained is best possible.

152. On some associated polynomials : *R.C. Singh Chandel,(Orai) and B. N. Dwivedi, (Atarra).*

To unify the study of several polynomial systems Chandel and Dwivedi [Publication De L' Institute of mathematique Belgrade (commnnicatcd)] introduced a general class of Polynomials and have discussed two special cases for

$$c_n = \frac{(q)_n}{n\,!} \quad \text{and} \quad c_n = \frac{(-1)^n}{n\,!}.$$

The motivation of this paper is to introduce the three sets of polynomials related to the particular case

$$(C-2xt+yt^2)^p \ exp\left[\frac{-r^r\, z\, t^r}{(C-2xt+yt^2)^r}\right]$$

$$=\sum_{0}^{\infty} E_n^p\,(2, x, y, z, r, r, C)t^n.$$

153. Generalized Van der Pol polynomials : *T. R. Prabhakar and Reva (Delhi).*

A general sequence of polynomials, which is an Appell set is studied with respect to the invertible shift-invariant operator $[\beta D(E+a)+\gamma(E+b)](\alpha D^m)^{-1}$

where $\alpha, \beta, \gamma, a, b$ are real numbers with $\alpha \neq 0$, m is a non-negative integer, D and E are differential and shift operators respectively. For this polynomial $Z_n(x\,;m: \ \alpha, \beta, \gamma\,; a, b)$, a generating relation, fundamental identity, complementary theorem, recurrence relations. integrals etc. are obtained. This polynomiai includes as special cases the Van der Pol polynomials along with the related polynomials of Bernouli and of Euler. The study uses the techniques of finite operator calculus

developed recently by G-C. Rota, D. Kahaner and A. Odlyzko.

154 On a new class of integral operators : *V.M. Bhise and K. Sarojini (Indore).*

In this paper we introduce a new class C_w of integral operators T_w defined by

$$T_w(f)(x) = \int_0^\infty w(y)\, K(xy) L(x/y)\, f(y)\, dy.$$

Three theorems for the operator and its adjoint are proved. Applications of the theorems are also given.

155 On inversion operator for the Post-Widder inversion operator : *R. P. Manandhar (Kirtipur).*

In earlier papers the author has defined a real inversion operator for the Post-Widder inversion operator by

$$\text{(1)} \quad p_{n,t}^{(\beta,\, v)}\left[L_{k,t}^{(\cdot)}[f]\right] = A(k,n) \int_0^\infty x^{v}\, Q_v^{(0,\, \beta+2n)}[1+2x^{-1}]\, L_{kt}^{(x)}[f]\, dx$$

where

$$\text{(2)} \qquad L_{k,t}^{(x)}[f] = \frac{1}{k!}\left(\frac{k}{t}\right)^{k+1} \int_0^\infty e^{-\frac{k}{t}xu}\, u^k\, \phi(u)\, du; \; x>0,\ t>0,$$

$A(k, n)$ is given by

$$\text{(3)} \qquad \frac{\Gamma(v+1)\, A(k,n)}{2\Gamma(k+1)} = \frac{e^{-2(k+n)}\, \Gamma(\beta+v+2n+1)}{[\Gamma(k-v)]^2\, \Gamma(\beta+2v+2n+1)}$$

and $Q_v^{(\alpha,\, \beta)}[x]$ is a Jacobi function of the second kind, defined by

$$\text{(4)} \qquad Q_v^{(\alpha,\, \beta)}[x] = \frac{2^{\alpha+\beta+v}\, \Gamma(\alpha+v+1)\, \Gamma(\beta+v+1)}{\Gamma(\alpha+\beta+2v+2)\, (x-1)^{\alpha+v+1}\, (x+1)^{\beta}}$$

$$ {}_2F_1\left[\begin{matrix} v+1,\ \alpha+v+1\ ; \\ \alpha+\beta+2v+2\ ; \end{matrix}\ 2(1-x)^{-1}\right]$$

and it has been shown under certain conditions that

$$(5) \qquad \lim_{n \to \infty} P^{(\beta,\, \nu)}_{n,\, t} \left[L^{(\cdot)}_{k,\, t} [f] \right] = \phi(t)$$

for almost all $t > 0$.

In this paper the inversion formula (5) is extended to generalised functions by interpreting the convergence in the weak distributional sense.

156. Satpute's first theorem on Laplacian operator : *Anilkumar Baburao Satpute, (Bombay).*

Abstract :

In Gauss's Divergence Theorem we have the relation between the normal component of a vector point function and the divergence of that with the surface and the volume Integrals respectively.

We now proved the same for the Laplacian of scalar function and the divergence of that. The theorem states as follows :

Satpute's First Theorem :

The surface integral of the divergence of vector function $\vec{F}$, taken over closed surface S, enclosing a volume V, is equal to the volume integral of the Laplacian of the scalar function F taken over the volume V.

$$\boxed{\iiint_V \nabla^2 F \, dV = \iint_S Div \, \vec{F} \, ds}$$

Satpute's second theorem :

The line integral of the vector function $\vec{F}$ taken over the curve C, enclosed by the surface S, is equal to the volume integral of the Laplacian of Scalar function F taken over volume V

$$\boxed{\iiint_V \nabla^2 F \, dv = \oint_C \vec{F} \, . \, \vec{dn}}$$

157. Some necessary conditions for the representation of certain generalized Laplace transforms of distributions : *G. L. N. Rao, (Jamshedpur).*

Among the several mathematicians who developed the generalizations of the classical Laplace Transform, the namesof Meijer, Boas, Erdelyi and Varma need to be mentioned first. The mathematical literature is full of numerous results regarding inversion, representation and other properties of these generalized Laplace Transforms.

Like inversion, representation theory is a very essential part of any transform theory. The representation problem for the conventional Laplace Transform was solved by Widder. The same problem for distributions was solved by L. Schwartz.

The author of the present abstract aims at an extension of certain necessary conditions for the representation of the generalizations of Laplace Transforms given by Meijer and Erdelyi, to a certain space of generalized functions.

158. The n-dimensional distributional k-transformation : *Sunil Kumar Sinha (Ranchi).*

The *K*-tranformation for ordinary function was first introduced by Meijer in 1940 and then by Boas in 1942 and Erdelyi in 1950-51. As a result of extension of integral transformations to distributions by A. A, Zemanian in the year 1966, he (Zemanian) himself generalized the *K*-transformation of order $\mu(-1/2\leqslant \text{Re}\mu\leqslant 1/2)$. Recently (1975) E. L. Koh developed n-dimensional case corresponding to a paper of Zemanian entitled "A Distributional Hankel Transformation SIAM J. 14 (1966), pp. 561-576". In present paper we develop n-dimensional case corresponding to a paper of Zemanian entitled "A Distributional K-transformation, SIAM J.14 (1966) pp. 1350-1365", using result of A. H. Fleming (Functions of several variables, Addison-Wesley, Reading, Mass, 1965) where applicable.

159. On certain genrealized q-Laplace and Stieltjes transforms : *Ramadhar Mishra, (Allahabad).*

In this paper, we define two q-analegues of a generalized Laplace transform due to J. M. C. Joshi by applying R. P. Agarwal's operater of q-fractional integration and study some of its general properties. Certain q-Stieltjes transforms, obtained as the first iterate of the two basic analogues of the classical Laplace transform introduced earlier by W. Hahn, have also been defined and some of their generalizations and general properties are studied.

160. On lacuanary interpolation for entire functions : *P. Bhattacharyya, V. Kala.*

In this paper the problem of (0, 1, 2...r, , r+2), (0, 3) and (0, 4) interpolation by entire functions of exponential type over the infinite interval $(-\infty, \infty)$ is considered.

We prove the following theorems.

Theorem 1. Given an arbitrary sequence of constants $a_{o,k}$, $a_{1,k}$, $a_{3,k}$ *where* $k=0, \pm 1, \pm 2, \ldots$, *then there exists an entire transcendental function of exponential type* 3σ *such that*

$$f\left(\frac{k\pi}{\sigma}\right) = a_{o.\,k},\ f'\left(\frac{k\pi}{\sigma}\right) = a_{1,\,k}\,,\ f'''\left(\frac{k\pi}{\sigma}\right) = a_{3,\,k}$$

which is bounded on $(-\infty, \infty)$. *However f is not unique.*

Theorem 2. Given an arbitrary sequence of constants

$$a_{o,k}\,,\ a_{1,k}\,,\ \ldots\ a_{r,k}\,,\ \ ,\ a_{r+2,k}\ \ ,\ k=0, \pm 1, \ldots$$

there exist no unique entire function f of exponential type $(r+2)\sigma$ *such that*

$$f\left(\frac{k\pi}{\sigma}\right) = a_{o,k}\,,\ f^{1}\left(\frac{k\pi}{\sigma}\right) = a_{1,k}\,,\ldots,\ f^{r}\left(\frac{k\pi}{\sigma}\right) = a_{r,k}$$

$$f^{r+2}\left(\frac{k\pi}{\sigma}\right) = a_{r+2\cdot k}$$

where f^{i} *denotes the* i^{th} *derivative of f.*

Theorem 3. Given arbitrary sequence of constants a_k *and* b_k, $k=0$, $\pm 1, \pm 2, \ldots$

there does not exist any entire function f of exponential type 2σ, *bounded on* $(-\infty, \infty)$ *such that*

$$f\left(\frac{k\pi}{\sigma}\right) = a_k \text{ and } f'''\left(\frac{k\pi}{\sigma}\right) = b_k\,.$$

or

$$f\left(\frac{k\pi}{\sigma}\right) = a_k \text{ and } f^{iv}\left(\frac{k\pi}{\sigma}\right) = b_k.$$

161. A class of expansions of hypergeometric functions : *R. P. Singal, (Ferozepore).*

The author, in his paper (Math. Student 38 (1970) 63-64) gave a number of expansions which generalize the result due to Verma (Math. Compt. Vol. XIX. no. 92 : p. 664, 1965). In this paper we have obtained an expansion or generalized hypergeometric function of n variables.

162. Some relations between basic hypergeometric functions of three variables and their contiguous funetions : *S. K. Prasad, (Giridih).*

In the paper I have deduced some relations between some of the basic hypergeometric functions of three variables and their contiguous functions.

163. Dynamical symmetry algebra of $_0F_1$ and Bessel functions : *Renu Jain aad B. M. Agrawal.*

In this paper use of the dynamical symmetry algebra (the maximal Lie-algebra of linear differential operators obtained on the basis of differential recurrence relations) of $_0F_1$ is made to derive two well known generating functions [Erdelyi : Vol. II ; p. 261.],

$$(i) \quad (z+t)^{-\alpha/2} J_\alpha\left[2(z+t)^{1/2}\right]=\sum_{n=0}^{\infty} z^{-\frac{1}{2}\alpha-\frac{1}{2}n} J_{\alpha+n}(2\sqrt{z})(-t)^n/n!$$

$$(ii) \quad (z+t)^{\alpha/2} J_\alpha\left[2(z+t)^{1/2}\right]=\sum_{n=0}^{\infty} z^{\frac{1}{2}\alpha-\frac{1}{2}n} J_{\alpha-n}(2\sqrt{z})\, t^n/n!$$

and two equally familiar d. r. relations (Rainville ; p ; 111)

$$(i) \quad zJ'_n(z) = -z\, J_{n+1}(z) + n\, J_n(z)$$

$$(ii) \quad zJ'_n(z) = z\, J_{n-1}(z) - n\, J_n(z).$$

The technique is based on introducing a variable 't' such that the linear differential operators do not refer to the parameters, thus facilitating their repeated operation.

The simple derivations lend us an insight into the fundamental role of hypergeometric functions in special functions theory besides illustrating the elegance of group theoretic approach.

164. Some new generating functions for Jacobi polynomials : *P. C. Jain (Bikaner).*

"In this paper we obtain some new generating functions for Jacobi polynomials. The results obtained generalize the results given earlier by Bailey, Manocha & Sharma and Brafman."

165. Some problems of linear homogeneous bibasic functional equations : *Wazir Hasan Abdi, (Cochin).*

An equation of the form

$$a(x)\,f(bx) + b(x)\,f(qx) + c(x)\,f(x) = 0$$

where a, b and c are functions analytic in some domain encircling the origin and p, q are unconnected bases. In earlier communications the problem of existence and methods of solution under various conditions were discussed. In this short note, some other properties like approximation and zeros of the function f will be discussed, also some open questions stated.

166. On the generalisation of Bessack's extension of opial inequality : *Madhavi Dighe and V. M. Bhise (Indore).*

In this paper an integral inequality, involving the Fourier type integral operator and its derivative is established. The result is the generalisation of Beesack's extension of Opial inequality. As an example an inequality involving the modified Bessel function is given.

167. On a functional equation and information measures with preference : *B.D. Sharma (The Univ. of the west Indies) and R. P. Singh, (Meerut).*

Guiasu gave a system of axioms for the weighted entropy which modifies the Shannon's system by introducing the concept of utility. Recently, Aggarwal and Picard found a bit more general result than these from Guiasu and Emptoz, Sharma and Manmohan using a probability space with two measures.

In this communication, we find a measure with the same kind of applications, as the measures of Belis and Guiasu, Aggarwal and Picard, and others. The additivity here used is called the weighted additivity of type (α, β). This type of addivity gives rise to a certain generalized functional

equation the solution of which gives rise to the information measure of the type (α, β). This is the generalized measure which in limiting case studies other new generalized measures with preference with applications in various sciences.

VIII MEASURE THEORY, PROBABILITY, INFORMATION THEORY

168. A universal binary code modifier : *H. L. Janwa and H. Subramanian.*

Let there be a binary block code of length n. We suggest a method of modification. A particular n tuple code word, with last l entries zeroes, is identified with a $(n-l+1)$ tuple word, where the first $(n-l)$ entries as it is from the original code word and $(n-l+1)$ th entry is a third symbol*. The binary channel is modified to handle such a third symbol * which is assumed to have transmission error negligibly small as compared with transmission errors of 0 and 1. The modified code words are transmitted over the modified channel and in the received$_*$ed vector, * is replaced by the number of zeroes it represents. The word is then decoded according to the original decoder. The reliability is improved because (i) of handling a symbol like * ; (ii) a significant part of the decoding failures can be detected since a decoded word must honour the meaning of *. Also the average length is decreased by a special assignment at the input according to source message probabilities. The transmission rate is accordingly increased.

169. The structure of certain invariant abundance distributions : *Jagannath K. Wani and Hing Po Lo (Calgary).*

Consider a population the individuals in which can be classified into groups. Let y, the number of individulas in a group, be distributed according to a probability function $f(y; \theta_0)$ where the functional form f is known. The random variable y cannot be observed directly and hence a random sample of *groups* cannot be obtained. Consider a random sample of N *individuals* from the population. Suppose the N individuals are distributed into S groups with $x_1, x_2, \ldots, x_S$ representatives respectively. The random variable x, the number of individuals in a group in the sample, will be a fraction of its population counterpart y and the distributions of x and y may not have the same functional form. Under what conditions the two random variables x and y will have the same functional form for their distributions ? The paper examines the class of series distributions for the existence of invariant distributions and discusses the relations between the values of the related series parameters.

170. Variational equations for a general and Gaussian channel with side Information : *Bhu Dev Sharma and Ved Priya (Delhi).*

Variational equations for continuous sources are known in the literature. In this paper we have obtained the variational equations by considering the side information about the source at the encoder and the decoder when

(1) a distortion criterion acts on the main channel only ;

(2) there are two distortion criteria one the main channel and the other on the side channel.

The paper studies convexity of the functions $R_{X/Y}(D)$ and $R_{X/Y}(D_1, D_2)$. The form of the variational equations for a Gaussian source considering distortion under squared error criterion on the main channel have been examined.

IX DIFFERENTIAL GEOMETRY, PROJECTIVE GEOMETRY, ANALYTIC GEOMETRY

171. Submanifolds of almost paracontact manifolds : *R.S. Mishra and J.P. Srivastava.*

I. Sato defined and studied a manifold with an almost paracontact structure. In the present paper we aim at obtaining the necessary and sufficient conditions, such *m-n* of an almost paracontact manifold possess induced almost paracontact and almost product Riemannian structure. We have also studied some properties of non-invariant, invariant submanifolds, when the enveloping almost paracontact manifold satisfies certain conditions.

172. Infinitesimal analytic holomorphically projective transformation in some Kahler spaces : *R.S. Sinha and H.C. Lal.*

The concept of infinitesimal holomorphically projective transformation (briefly HP-transformation) has been introduced by Ishihara, S. (Tohoku Math. J. 9(1957), 173-297) and analytic holomorphically projective transformation in Kahler manifolds has been studied by Tachibana, S. and Ishihara, S. (Tohoku Math, J. 12(1960), 77-101). In the present paper, the authors have studied analytic HP-transformation for recurrent and symmetric Kahler spaces (briefly RK_n and SK_n) and obtained the following important theorems :

Theorem 1. If a RK_n admits an infinitesimal analytic HP-transformation then one of the following must hold good :

(i) A_1 is a null vector. i.e. $L_v\, a_1 = (n-2)\rho_1$

(ii) $P^r_{kji}\ \rho_r=0.$

Theorem 2. An Einstein RK_n can not admit an infinitesimal analytic HP-transformation satisfying $\rho_r P^{\ r}_{\ kji} \neq 0$.

Theorem 3. In a RK_n admitting an infinitesimal analytic HP-transformation satisfying $\rho_r P^{\ r}_{\ kji} = 0$, Lie-derivative of recurrence vector, associated vector of HP-transformation and its transform all are Ricci-directions.

Theorem 4. A non-affine infinitesimal analytic HP-transformation satisfying $\rho_r P^{\ r}_{\ kji} = 0$ always maps an Einstein RK_n into an Einstein K_n

173. Full collineation group of Foulser's flag transitive plane of order 25 : *M.L. Narayana Rao and K. Kuppu Swamy Rao (Hyderabad).*

A collineation group of a finite affine plane π of order n is flag transitive on π, if it is transitive on the incident point-line pairs or flags of π. D.A. Foulser (1964) has determined the flag transitive collineation groups of finite Desargusian affine planes and constructed two non-Desargusian planes of order 25 and shown that his two planes of order 25 and the nearfield plane of arder 9 have flag transitive collineation groups. Flag transitive affine planes were discovered and reported by several authors. The aim of this paper is to determine the full collineation group of one of Foulser's flag transitive affine planes.

174. Full collineation group of Foulser's flag transitive plane π' of order 25 : *K. Satyanarayana and K. Kuppuswamy Rao (Hyderabad).*

A finite affine plane π is flag transitive if it has a collineation group, which is transitive on the incident pointline pairs or flags of π. D.A. Foulser (1964) has determined all flag transitive groups of finite affine planes and constructed two flag transitive planes π and π' of order 25 (D.A. Foulser, Solvable flag transitive affine planes, Math. Z., 86 (1964), 191-204) and shown that his two planes of order 25 and the nearfield plane of order 9 have flag transitive collineation groups. Since then the study of flag transitive planes has gained considerable interest. Recently, M.L. Narayana Rao and K. Kuppuswamy Rao have determined the full collineation group Foulser's flag transitive affine plane π of order 25 (to appear, Houston. J. Math). The aim of this paper is to determine the full collineation group of the other flag transitive plane π' of order 25.

175. A weaker set of axioms for a finite projective plane : *P. Srinivasan (Madurai).*

We prove, in this paper, the following Theorem, which gives a weaker set of axions for a finite projective plane of order n.

Theorem.

Let $\pi=(P, \mathbf{L}, I)$, where P is a set, called the set of points of cardinality n^2+n+1 for some $n\geqslant 2$, $\mathbf{L}$ is a set called the set of lines and I is a relation between the set of points and the set of lines called the incidence relation, satisying the following axioms.

1. (a) $\forall\ A,B\in P$, $A\neq B$ there exists atmost one line $l\in\mathbf{L}$ such that $A\ Il$ and $B\ I\ l$.

   (b) There exists a line $\bar{l}\in\mathbf{L}$ such that, $\forall\ A,B\in P(A\neq B)$ and $A\ I\ \bar{l}$, there exists a line $l\in\mathbf{L}$ such that $A\ I\ l$ and $B\ I\ l$.

2. There exists a $YI\bar{l}$ such that, $\forall\ l_1, l_2\in\mathbf{L}$ with $YI\ l_1$ there exisis $P\in P$ such that PIl_1 and PIl_2.

3. There exists a line $m\in\mathbf{L}$ such that YIm and $|\ m\ |=n+1$.

Then π is a projective plane of order n.

We also give counter examples to show that the above theorem is not valid if either the hypothesis on the cardinality of P or axiom 3 is dropped.

As a Corollary we also deduce Theorem 5.4 of D.R. Hughes (Projective Planes) on Planner Ternary Rings of finite planes.

176. On the semigroup of degenerate projectivities of a projective space : *K.S.S. Nambooripad and E. Krishnan (Kariavattom).*

If V is a vector space over a division ring D, a degenerate projectivity of the projective space $P=P(V)$ is a partial transformation on P induced by a semilinear endomorphism of V. In this paper we show that the set $T(P)$ of all degenerate projectivities of P is a fundamental regular semigroup. ['Structure of regular semigroups I', *K.S.S* Nambooripad. Memoirs Amer. Math. Soc. Vol. 22 No. 224, 1979] and that the semigroup $S(V)$ of all semi-linear endomorphisms of V is a uniform $\mathbf{H}$-coextension of $T(P)$ by the group D^* (the group of non-zero elements of D). Further, the semigroup of all degenerate collineations of P is a full, normal subsemigroup of $T(P)$. We also describe the inductive groupoid [loc. cit] of $T(P)$ in terms of the groupoid of projectivities of subspaces of P and the dual geometry P^*.

177. On left distributive law in ptrs, with zero, of a translation plane : *B. Maheswari, L. Nagamuni Reddy and K. Sitaram (Tirupati).*

Recently some studies have been made on planar ternary rings (PTRs)

with zero, of translation planes and Desarguesian planes in terms of elations and homologies respectively. In this paper we characterize the left distributive law in a PTR with zero by means of elations. The main results are as follows :

Theorem 1. A translation plane π is (Y,OY)-transitive if and only if the equation $ma+mb=mc$ either has only the trivial solution or it is fulfilled identically.

Theorem 2. If π is a translation plane and (S,T) is its PTR, then left distributive law $ma+mb=m(a+b)$, for all m,a,b in S holds in S if and only if the map α defined by $(m,n)\alpha=(m,ma+n)$, $(b)\alpha=(a+b)$ is a (Y,OY)-elation.

178. **The tetrahedron the feet of whose altitudes are coplanar** : *Kesiraju Satyanarayana* (*Hyderabad*).

Let $V(F)$ be the volume of the tetrahedron formed by F_i, the feet of the altitudes of a tetrahedron $(A)\equiv A_1A_2A_3A_4$ of volume V. Let h be the length of the altitude from *any* vertex (the triangle opposite and the F_i on it being named $\triangle A_1A_2A_3$ and F). Let

$$\phi_r\equiv[\sum_{r=1}^{3} p_r^2\zeta_r^2)-2(p_1p_2\zeta_1\zeta_2+p_1p_3\zeta_1\zeta_3+p_2p_3\zeta_2\zeta_3]h^4$$

$$+2\zeta_1\zeta_2\zeta_3(\sum_{r=1}^{3} p_r\zeta_r)\,\delta^2h^2-3\zeta_1^2\quad\zeta_2^2\quad\zeta_3^2\,\zeta^4,$$

where δ is twice the area of $\triangle A_1A_2A_3$; p_1,p_2,p_3 are the powers of $A_1A_2A_3$ *w.r.t.* the circles on A_2A_3, A_3A_1, A_1A_2 as diameters; and ξ_1, ξ_2, ξ_3 are the areal coordinates of F *w.r.t.* $\triangle A_1A_2A_3$.

If F_i is the area of the face triangle of (A) opposite A_i it is shown that

(1) $16\,F_1^2\,F_2^2\,F_3^2\,F_4^2\,V(F)/V=\phi\,F_4^4$;

(2) *F_i are coplanar if and only if $\phi=0$*

The case of non-distinct F_i^s is also studied.

(2) is a generalisation, in geometrical form, of an analytical result when A_i are (0,0,0), (1,0,0), (0,1,0), (x,y,z) communicated to the author by Professor Rudolf Fritsch of Universitat konstant in his letter of January 15, 1980.

179. **Equilateral triangles inscribed in a given triangle** : *Kesiraju Satyanarayana* (*Hyderabad*).

Let $\triangle ABC$ be the given triangle with incentre I, circumradius R and

area $\triangle$; $\triangle A'B'C'$ any equilateral triangle (with A',B',C' on the sides or sides produced of ABC) its centre being O and length of its side being l; ζ_1,ζ_2,ζ_3 the areal coordinates of a point *w.r.t.* $\triangle ABC$. Then (i) if $\triangle ABC$ is equilateral, $O\equiv I$ for all $\triangle^s A'B'C'$; and the minimum value of l is $\frac{a}{2}$

(2) (i) in general, there are two sets of $\triangle^s A'B'C'$—$\triangle^s$ $A_1' B_1' C_1'$, $\triangle^s$ $A_2' B_2' C_2'$, O,l being replaced by $O_1,O_2;l_1,l_2$: (ii) the locus of $O_1[O_2]$ is the straight line

$$\zeta_1 \frac{\sin\left(A - \frac{\pi}{3}\right)}{\sin A} + \zeta_2 \frac{\sin\left(B - \frac{\pi}{3}\right)}{\sin B} + \zeta_3 \frac{\sin\left(C - \frac{\pi}{3}\right)}{\sin C} = 0$$

$$\zeta_1 \frac{\sin\left(A - \frac{\pi}{3}\right)}{\sin A} + \xi_2 \frac{\sin\left(B + \frac{\pi}{3}\right)}{\sin B} + \xi_3 \frac{\sin\left(C + \frac{\pi}{3}\right)}{\sin C} = 0$$

(iii) the minimum value of l_1, $[l_2]$ is

$$\frac{2\triangle}{R}\left[1+8 \cos\left(A - \frac{\pi}{3}\right) \cos\left(B - \frac{\pi}{3}\right) \cos\left(C - \frac{\pi}{3}\right)\right]^{-\frac{1}{2}},$$

$$\left[\frac{2\triangle}{R}\left\{1+8 \cos\left(A + \frac{\pi}{3}\right) \cos\left(B + \frac{\pi}{3}\right) \cos\left(C + \frac{\pi}{3}\right)\right\}^{-\frac{1}{2}}\right].$$

180. Generators of points whose coordinates are cyclic permutations of linear functions of the coordinates of the vertices of a triangle and from which perpendiculars to the sides of the triangle are concurrent at a specified point : *M.N. Ramakrishna Pillai (Trivandrum).*

Let (x_1,y_1) (x_2,y_2) (x_3,y_3) be the coordinates of the vertices ABC of a triangle ABC and the coordinates of the point of concurrence of perpendiculars from the four sets of symmetric points,

1	2	3	4	
(mx_1,my_1)	$\{n(x_2+x_3),n(y_2+y_3)\}$	(py_1,px_1)	$\{q(y_2+y_3),q(x_2+x_3)\}$	to BC
(mx_2,my_2)	$\{n(x_3+x_1),n(y_3+y_1)\}$	(py_2,px_2)	$\{q(y_3+y_1), q(x_3+x_1)\}$	to CA
(mx_3,my_3)	$\{n(x_1+x_2), n(y_1+y_2)\}$	(py_3,px_3)	$\{q(y_1+y_2), q(x_1+x_2)\}$	to AB

be (mS_1,mT_1) (nS_2,nT_2) (pS_3,pT_3) and (qS_4,qT_4) respectively where m,n,p,q, are real numbers (see pages 80 and 81 of the Mathematics student for 1973). If (g,h) are the coordinates of any point it can be expressed in 6 ways as a linear combination of any two of (mS_1,mT_1) (nS_2,nT_2) (pS_3,pT_3) and (qS_4,qT_4). The 12 values of (m,n,p,q) so obtained give six sets of symmetric points from which perpendiculars to the sides of the triangle are con-

current at (g,h). By a suitable linear combination of multiples of any two of these six sets of symmetric points we can obtain values of m,n,p,q which will generate any number of symmetric points from which perpendiculars to the sides are concurrent at (g,h). The sets of points so obtained fall into five classes in four of which the coordinates of one of the four symmetric points is absent and one in which the coordinates of all the symmetric points are present.

X APPLICATIONS OF MATHEMATICS

181. On dispersion flow in absorbing porous media : *Mohd A. A. Ansari (Varanasi).*

The dispersion flow in absorbing porous media is studied when the input concentration varies exponentially with time. The Laplace transform is used to obtain the solution to the dispersion problem withln homogeneous, istropic and semi infinite porous media in unidirectional flow fields. he different types of variations in concentration have been graphically dis cussed.

182. Flow of a dusty gas between two oscillatting plates : *Evelyn Chandrasekharan (Chromepet).*

In this paper the problem of flow of an incompressible viscous dusty gas, induced by two infinite parallel flat plates oscillating in their own planes, is considered when an oscillatory body force (having the same frequency as that of the plates) is applied in the direction of motion of the plates. The gas is assumed to contain uniform distribution of dust particies. The effect of the dust is characterized by two dimensionless parameters viz. the mass concentration of dust 'λ' and the dimensionless relaxation time 'T' which is proportional to the rate at which the velocity of a dust particle adjusts to changes in the gas velocity and depends upon the size of the individual particles. Using the technique adopted in the classical Stokes' second problem, detailed analytical expressions for the velocity fields of the fluid and dust particles, volume flow rates and stress at the plates have been derived. It is found that the fluid velocity and dust velocity oscillate in time with the same frequency as that of the plates or the body force (but with a phase difference). Velocity profile curves are drawn for various situations and the influence of the body force on the flow of the fluid and dust particles is investigated. Also the effects of changes in 'λ' and 'T' on the fluid and dust particles are studied. Lastly the times at which the fluid and dust

particles in the mid-region are momentarily at rest and the times at which the volume flow rates and stress at the plates vanish are determined.

183. Boundary layer flow of a power law fluid with suction : *Som Prakash Singh (Banaras).*

Here, we are taking laminar compressible boundary layer flow of a power law fluid due to a step change in surface flux over a semi-infinite flat plate with suction at the surface. Skin-friction and wall temperature gradient are obtained, for different values of Prandtl numbers, by the use of perturbation technique.

184. Unsteady flow of a dusty viscous incompressible fluid through a circular pipe : *S. Vidya (Bangalare).*

The flow of dusty viscous incompressible fluid through a circular pipe is investigated in this paper in two cases namely (*i*) When pressure gradient varies linearly with time & (*ii*) When it varies harmonically with time. These investigations coroborate with the experimental observations and prove that the dusty fluids move slower than the clean fluids ; also whenever the dust particles become very fine or after an infinite lapse of time the influence of the dust particles in the viscous fluid become negligible.

185. Mhd flow through a porous medium bounded by an oscillating porous plate : *A. K. Singh (Varanasi).*

A solution in the closed form has been obtained for the flow of a viscous incompressible fluid of small electrical conductivity in a porous medium near an oscillating infinite porous flat plate in the presence of a transverse magnetic field of uniform strength ; fixed relative to the fluid. The graphical representations show that the porous medium decreases the velocity profiles.

186. Symmetric marching technique (SMT) for the efficient solution of discretized poisson equation on non-rectangular regions : *Mohan K. Kadalbajoo and K. K. Bharadwaj (Kanpur).*

In this paper, two methods based on the symmetric marching technique (SMT) have been presented for the solution of discretized Possion equation on non rectangular regions. The method I illustrates the direct adoptation of SMT to irregular geometries. In method II, an efficient implementation of the capacitance matrix method has been considered using SMT. The favourable properties of SMT, to solve Poisson equation subject to several

right hand side functions and different boundary conditions without extra computational effort, have been exploited for the fast generation of the capacitance matrix. Numerical results, of the model problems solved, have been presented.

187. Slow steady flow of a couple stress fluid generated by the rotation of two concentric spheres about distinct diametres : *S.K. Lakshmana Rao,*

K. Venkatapathi Raju and T.K.V. Iyengar (Warangal).

The paper deals with the steady flow of an incompressible couple stress fluid generated by the rotation of two concentric spheres rotating about two distinct diameteres. The flow velocity q can be expressed in the form

$$q = \omega_1 F_1(r)\, e_r \times e_{\omega_1} + \omega_2\, F_2(r)\, e_r \times e_{\omega_2} \tag{1}$$

where e_{ω_1}, e_{ω_2} are the unit axial vectors along the diameters about which the spheres rotate, (ω_1, ω_2) are the angular speeds of rotation, e_r is the unit radial vector and r is the radial distance from the centre. The functions $F_i\,(r)$ $(i=1, 2)$ are obtainable from the differential equation

$$D(D-(\lambda^2/a^2))\ F(r)=0 \tag{2}$$

where

$$D= \frac{d^2}{dr^2} + \frac{2}{r}\ \frac{d}{dr} - \frac{2}{r^2} \tag{3}$$

and (λ^2/a^2) is the couple stress parameter. The boundary conditions on $F_i\,(r)$ arise from the hyperstick condition at the two spherical boundaries $r=r_1$, $r=r_2$ and are separately expressible in terms of the functions F_1 and F_2. The solutions $F_1\,(r)$ are seen to be

$$F_i(r)=a_i r+b_i r^{-2}+r^{-1/2}(c_i I_{3/2}(\lambda r/a)+d_i K_{3/2}(\lambda r/a)) \tag{4}$$

$(i=1, 2)$ and the constants are determined from a linear system of algebraic equations. The couple on the spheres is evaluated by a standard method and is expressible in the form

$$C=8\pi\mu(\omega_1 e_{\omega_1} - \omega_2\, e_{\omega_2})b_1 \tag{5}$$

Numerical studies have been undertaken to examine the variation of the couple with respect to the paraameter λ as well as the ratio of the radii of the spheres.

188. Stability of viscous flow between rotating cylinders with transverse pressure gradient: *A.K. Ganguli and Mrs. M.S. Rao (Patna).*

In the theory of hydrodynamic stability, one often has to encounter characteristic value problems posed by differential equations of relatively high order which do not allow variational formulations in terms of expressions for the characteristic values as ratios of positive—definite integrals. Roberts (1960), however. suggested that in many of these instances one may be able to define systems which are in some sense adjoint to the ones considered, and in terms of which, one may, indeed, obtain the solutions to the basic problems by variational methods. In this paper, this method of adjointness is employed in a study of the stability of viscous flow between coaxial rotating cylinders in the presence of a transverse pressure gradient.

189. Peristaltic transport of a power-law fluid with variable consistency : *J.B. Shukla and S.P. Gupta (Kanpur).*

Effects of the consistency variation on the peristaltic transport of a non-Newtonian power-law fluid through a tube have been investigated by taking into account the existence of a peripheral layer. It is shown that the flow flux, for zero pressure drop, increases as the amplitude of the peristaltic wave increases but it decreases due to the pseubo-plastic nature of the fluid. It is also shown that, for zero pressure drop, the flux does not depend on the consistency of peripheral layer fluid while the friction force decreases as this consistency descreases. However, for non-zero pressure drop, the flux increases and the friction force decreases as the consistency of the peripheral layer fluid decreases.

190. The effect of temperature on edge waves in plates : *T. Ramesan & N. Bhoje Gowda (Bangalore).*

In this paper edge waves in plates are discussed together with the various temperature. The case of flat plate of specified width and infinite length is solved and the frequency equation for the propagated waves is obtained by assuming the plate to be thin.

191. Thermo elastic excited stresses waves : *T. Ramesan & N. Bhoje Gowda (Bangalore).*

In this paper we have discussed in detail the Thermo Elastic Excited Stresses Waves. The solution is obtained and in the limiting case whenever the surface stresses free we obtained thermoelastic Raleigh waves.

192. Thermoelastic waves due to the diffraction of pressure wave : *A.B. Kumar (Singhbhum).*

Coupled thermoelastic wave problem for an infinitely long cylindrical cavity in infinite homogeneous and isotropic elastic media under a plane shock wave with constant force impact on the boundary of the cavity has been considered, The stress field and the temperature field due to the diffraction of the incoming shock waves have been considered.

193. Wave motion in a falling film of liquid of variable viscosity : *S.K. Roy and J. P. Agarwal (Kharapur).*

The problem of wave motion in falling liquid films of variable viscosity is investigated. Such a case arises in the flow of a condensate down a wall with a linear temperature gradient through the film. The viscosity μ is taken to be governed by the relation

$$\mu_h \, exp\left(-\alpha \frac{y}{h}\right)$$

where μ_h is in the viscosity at the free surface, h the thickness of the film, and α a constant signifying how rapidly μ changes with the increase in y, y being measured in the direction of h. The non-dimensional wave number, wavelength and Weber number have been obtained. It is found that for all values of α in $0 \leqslant \alpha \leqslant \beta$, where $\beta \approx 1.4$, the wave number and Weber number exist if wave celerity <3. However, for values of $\alpha > \beta$, the wave number and Weber number exist only if wave celerity >3. The wavelength becomes infinite, if the Weber number is zero or infinite, and hence it attains a minimum value corresponding to a certain value of the Weber number for each α. The amplitude is found to increase with decrease in the wave celerity.

194. Stresses in an eccentric annulus under concentrated forces : *Mohammed Mukhtar Ali and Aqeel Ahmad (Aligarh).*

The problem of a thin isotropic elastic plate in the form of an eccentric annulus under concentrated forces has been solved. The plate is in equilibrium under two equal standard concentrated forces applied to the outer boundary where it has maximum and minimum thickness. Stresses have been found out on the outer boundary, and stress intensity factor is studied graphically for different radii of the outer boundary.

195. Effect of plate temperature oscillations on free convection boundary layers along a vertical plate when the plate temperature is non-uniforms : *P. Singh and V. Radhakrishnan (Kanpur).*

In this paper the laminar free convection flow and heat transfer along a semi-infinite vertical plate is analysed when the plate temperature T_p takes the form $T_p = T_w + \alpha(T_w - T_\infty)\,\mathrm{Sin}\,\omega t$ where $0 \leqslant \alpha < 1$ and $T_w - T_\infty = CX^n$. Thus the plate temperature consists of a basic steady distribution T_w with a super-imposed oscillatory distribution $\alpha(T_w - T_\infty)\mathrm{Sin}\,\omega t$. The main difference between this and the earlier works is that here the magnitude of oscillation α is not required to be small and the plate temperature is not uniform. A regular expansion is obtained for small values of the frequency parameter ϵ and the resulting equations are solved numerically. The skin friction and the rate of heat transfer from the wall are calculated for various values of α, n and Prandtl number σ. We observe that the maximum value of skin friction and heat transfer increases with α while the minimum value decreases as α increases.

196. Approximate solutions of laminar incompressible boundary layer equations with viscous dissipation : *P. Singh and S. Antony Raj (Kanpur).*

This paper deals with a new variational procedure based on Gyarmati's 'Governing Principle of Dissipative Processes' to solve analytically the boundary layer equations of two-dimensional laminar incompressible constant property fluid flow over a plane wall. The wall temperature distribution is assumed to satisfy the power law $T_w - T_\infty = Cx^n$. The velocity and temperature distributions inside the boundary layer are approximated by fourth degree polynomial functions. The principle is formulated and the flow field is determined by solving continuity and momentum equations. With the help of the obtained flow field and the principle, the energy eqation in which viscous dissipation term is included, is solved subsequently. The Euler-Lagrange equation reduces to a simple polynomial equation in thermal layer thickness whose coefficients depend on n, velocity boundary thickness, Prandtl number P and Eckert number E. The local heat transfer is calculated for different values of E when P and n lie respectively in the ranges $0 < P < \infty$ and $-2.5 \leqslant n \leqslant 4$. The present method is the best among all existing approximate methods because it is very general in

nature and the results are within 2 percent error. The partial differential equations are reduced to polynomial equations which enable engineers rapid calculation of heat transfer.

197. **Effects of dispersion on stability of two interacting species system in a patchy habitat :** *Sunita Verma and J.B. Shukla (Kanpur).*

In this paper the linear and non-linear stability of two interacting and migrating species systems have been investigated in a two dimensional patchy habitat. It has been shown that otherwise unstable equilibrium state can become stable with dispersive and convective migration.

The effects of dispersion on the criteria and the region for stability have been investigated.

198. **Effects of dispersive migration on stability of two species-system in two-dimensional habitates under non-uniform reservoir condiions :** *V.P. Shukla and J.B. Shukla (Kanpur, .*

A general population model of two interacting and dispersing species In two-dimensional habitats has been considered to study the local stability of an equilibrium state of the system and the conditions for stability derived. It has been shown that due to non-uniform reservoir boundary conditions, the distributions of the species form spatial patterns which depend upon both longitudinal and transverse directions in contrast to the case of uniform boundary conditions where these distributions depend only upon the longitudinal direction as in the case of one-dimensional linear habitat.

199. **Surface displacements over a thermodiffusive halfspace :** *K. S. Harinath (Bangalore).*

The expressions for the displacement components over the free surface of a thermodiffusive halfspace are obtained, by assuming the concentration over the surface to depend on a simple harmonic time-factor. In the case of thermal insulation, the solutions are further analysed. It is seen that shear waves are neither influenced by temperature nor by diffusion. In case the surface concentration is zero, the classical results of Rayleigh for waves propagated over an elastic halfspace are reproduced.

200. **High order methods for the numerical solution of two-point boundary value problems :** *J.R. Cash (London) and A. Singhal (Delhi).*

In a recent paper Cash and Moore have given a fourth order formula for

the approximate numerical integuation of two-point boundary value problems in O.D.E.S. The formula presented was in effect a "one-off" formula in that it was obtained using a trial and error approach. The purpose of the present paper is to describe a unified approach to the derivation of high order formulae for the numerical integration of two poiat boundary value problems. It is shown that the formula derived by Cash and Moore fits naturally into this framework and some new formulae of orders 4,6 and 8 are derived using this approach. A numerical comparison with certain existing finite difference methods is made and this comparison indicates the efficiency of the high order methods for problems having a suitably smooth solution.

201. **Perturbations of the elements of artificial satellite due to the oblateness of the earth** : *K. B. Bhatnagar and Z.A. Taqvi* (*Delhi*).

The subject of this paper is the extension of the work done by proskurin and Batrakov (1960). Expressions are found for calculating the perturbations of the elements of artificial earth's satelite upto the second power of Earth's oblateness. A numerical example is also taken showing the contribution of the second power of Earth's oblateness to different perturbations.

202. **Variance Bounds for an inverse binomial estimator** : *Ashok Sahi* (*Roorki*) *and Ajit Sahai* (*Lucknow*).

Best (1974) found the variance of the minimum variance unbiased estimator of the Bemoulli parameter with an inverse sample. Noting its intricacy Mikulaki and Smith (1976) found bounds on this variance. Sathe (1977) and Ray and Sahai (1978) developed closer bounds on it. In this paper still closer lower bound on the variance is achieved using generalised Jenson's inequality.

203. **Optimal inputs for approximate linear system in Hilbert spaces** : *R.K. S. Rathore* (*Kanpur*).

We consider the problem of determining on optimal input given only an approximate linear model of an actual input-output system. The optimality criterion is the minimization of the maximal possible error in the output. Under the assumption that the input and the output spaces can be modelled as Hilbert spaces, complete results on the existence, uniqueness, and characterization of optimal inputs are obtained. The results culminate in an unconditionally convergent iterative algorithm for optimal inputs.

204. **Dulity theorems and an optimality condition for non-differentiable convex programming :** *P. Kanniappan and M.A. Sundaram Sastry (Madurai).*

In this paper, we study the following pair of problems :

Problem (P) : Minimize $f(x)$ subject to $G(x) \leqslant 0$ and $x \in A$.

Problem (D) : Maximize $f(x) + \langle z^*, G(x) \rangle$ subject to

$$z^* \geqslant 0,\ x \in A \quad \text{and} \quad 0 \in \partial f(x) + z^* o \partial G(x) + N(x/A).$$

Here f is a continuous convex functional defined on a locally convex space X and G is a continuous convex operator, which is regularly subdifferentiable on A, a convex subset of X, defined on X into another locally convex space Z having a closed convex cone H defining a partial ordering in Z.

We prove a theorem of Fritz-John type.

Theorem 1. If x_0 is optimal solution of (P), then there exists $\lambda_0 \geqslant 0$, $z_0^* \in H^*$, not all zero such that $0 \in \lambda_0 \partial f(x_0) + z_0^* \, o \partial(x_0) + N(x_0/A)$ and $\langle z_0^*, G(x_0) \rangle 0$.

Theorem 2. If we assume in theorem 1, the Slater's constraint qualification, the above conditions are both necessary and sufficient for x_0 to be an optimal solution of (P).

Using the above Theorem 2 of Kuhn-Tucker type, we prove a duality theorem between problems (P) and (D).

Theorem 3. If x_0 is an optimal solution of (P) then there exists an z_0^* such that (x_0, z_0^*) is Optimal for (D). Further the two problems have the same extremal values.

We also obtain a converse duality theorem assuming that the objective function f is strictly convex at the solution point x_0 of (P).

205. **Reduced matrix elements of a coupled tensor in a subgroup basis :** *K. Bharathi (waltair).*

The reduced matrix elements of a coupled tensor between (*i*) uncoupled kets and (*ii*) coupled kets are derived in a group-subgroup basis using the

phase choices of Bulter (1975).

206. Complementary variational principles for journal bearing : *M.A. Gopalan (Tiruchirapalli).*

Bearings have an important role in machines involving moving members and are generally fed with incompressible or compressible lubricants. In this paper, the equation governing the flow of an incompressible lubricant in a journal bending, namely, the Reynolds equation

$$\frac{\partial}{\partial\theta}\left(H^3\frac{\partial p}{\partial\theta}\right)+\frac{\partial}{\partial y}\left(H^3\frac{\partial p}{\partial y}\right)=\frac{6\mu UR}{c^2}\frac{\partial H}{\partial\theta} \qquad (1)$$

subject to the boundary conditions

$$p(\theta,-L/2R)=p_s, \quad p(\theta,L/2R)=0$$

$$p(-\pi, y)=p(\pi, y), \ \frac{\partial p}{\partial\theta}\left(-\pi, y\right)=\frac{\partial p}{\partial\theta}\left(\pi, y\right) \qquad (2)$$

is derived from a variational principle, where the symbols have their usual meaning. Further, complementary variational principles have been developed which enable one to find the upper and lower bounds on the load capacity of the journal bearing without having recourse to finding the series solution or numerical solution of the differential equation. Bounds are also obtained for the case of axial feeding and the agreement with the earlier known solutions is found to be good.

207. Continued fractions and certain computational schemes : *B. M. Nayar.*

In applied problems it is often necessary to arrive at analytic solutions e. g. Taylor's series, rational functions or Mittag-Lcffler type expansions suiting the requirements. Continued fractions, representing yet another important form have comparatively more advantages.

The recurrence relations that exist between the successive approximants of continued fractions make them computationally more efficient. They bear a striking similarity with the successive approximants of the characteristic polynomial, or the polynomial generated by Lanczos minimized iteration or the Bernoulli sequences formed by the quotient-difference algorithm.

Interesting canonical decompositions are obtained when Schwarz's constants are employed as coefficients of the polynomial.

The motivation is to employ continued fractions for the solution of functional equations arising in the boundary value problem, particularly the first boundary value problem, a special case of the Robin-Poincare problem, leading to the Fredholm-Poincare integral equation.

The aim is to develop an existence and uniqueness theory best suited to numerical computations.

208. Forecasting techniques for economic data : *T.S. Ravisankar* (*Pilani*).

Forecasting the values of the relevant economic parameters into the future time scale is an integral component of any planning process, be it at the national level or the corporate level. The success of a plan would thus depend, to a large extent, on the efficiency of the forecasts based on which the shortfalls in the variables have been estimated and operational plans have been formulated to achieve a desired objective. Various statistical techniques, especially multiple regression and time series analysis have been used in this context. The forecasts obtained by these techniques in general tend to differ from the actuals significantly due to factors like policy changes, seasonal fluctuations, production shortfalls etc. While policy changes and like have to be subjectively evaluated, other factors affecting the forecasts could be taken care of by improved modelling of the situation. Since multiple regression techniques, depend on identifying empirical relationships between various parameters, their usage becomes difficult in many economic contexts. As such it would be advantageous to model the behaviour of an economic parameter by stochastic process representing the time series data of the past. In this setting, it is the contention of the present paper that a combination of the ARIMA models of Box/Jenkins together with "discounted least sequares" regression would prove a suitable technique. More specifically we consider a live banking data to support our contention, by making a comparative study of pure ARIMA models and combined models. The paper also discusses possible multiple time series forecasts to take care of inter-relationships between the parameters.

209. The theory of constanty through relativity : *S. S. Weispeute* (*Jalgaon*).

On the inconsistency of quotient module structures : *G. Ekanathan (Bangalore).*

A note on radicals in lattice ordered anti flexible rings : *M. C. Bhandari and A. Radhakrishna (Kanpur).*

A characterization of prime radical in near rings : *V. Sambasiva Rao and Bh. Satyanarayana (Nagarjuna Nagar).*

Noetherian s-space and primary decompositions : *A. Sita Ram Murti (Bhimavaram).*

Multipliers in hypergroups : *Om Prakash Agarwal (Delhi).*

Diophantine representation of Pell-numbers : *H.V. Krishna (Manipal).*

Nearly continuous functions : *R. Sarma and S. Katragadda (Nagarjunanagar).*

A note on S-closed spaces : *A.R. Singal aud R.C. Jain (Meerut).*

On weighted translation semigroups : *B.S. Yadav and S. Chatterjee (Delhi).*

Gelfond-Mazur theorem for quaternionic Banach triple systems : *V. Ramaswamy (Madras).*

On the derivative of a meromorphic function with maximum defect : *S.K. Anand (Delhi).*

Green's function and certain variable regions : *A. Kapur (Delhi).*

L_p-approximation by linear combinations of modified Bernstein polynomials : *T.A.K. Sinha (Kanpur).*

Initial and final-value theorems for the distributional Whittaker transform : *Kokila Sundaram (Jamshedpur).*

On characterization of generalized measure of entropy : *D.S. Hooda and P.N. Arora (Hissar and New Delhi).*

Longitudinal dispersion in saturated porous media : *Rajendra Singh (Varanasi).*

Effect of perturbed potentials on stability of libration points in the restricted problem : *K.B. Bhatnagar and P.P. Hallan (Delhi).*

Effect of a very strong magnetic field on acceleration covariance in MHD turbulance : *N. Kishore and T. Dixit (Varanasi).*

Qualitative theory of reaction-diffusion equations : *D.R.K. Sastry (Hyderabad).*

Treasure problem : *D.M. Patel (Ahmedabad).*

Brahma's problem : *D.M. Patel (Ahmedabad).*

Regular partial bands : *S. Premchand (Calicut).*

Idempotent generated free semigroups : *S. Premchand (Calicut).*

Independence of axioms for biordered sets : *S. Premchand (Calicut).*

Generalised Rodrigue's formule for classical polynomials and related operational relations : *P.N. Srivastava and Amar Singh (Jhansi).*

Euler polynomials of second kind and order α **:** *T.R. Prabhakar and Sharda Gupta (Delhi).*

The prime spectrum of a Lie algebra : *P. S. Rama and P. Dorai (Madras).*

Fixed point theorems in left (right) sequentially compact quasi-gauge space : *Ramesh Shrivastav, Tej Onkar Singh and N.P.S. Bawa (Rewa).*

On posemiring fuzzy languages : *R.N. Lal and B.P. Sinha (Bhagalpur).*

An expansion formula for multivariable H-function in terms of Bessel polynomials : *Awadhesh K. Singh and Y.N. Prasad (Varanasi).*

Minimal prime ideals in duo-semigroups : *S. Sribala and G. R. Venkataraman (Madras).*

On the kinetic theory for fluid turbulence : *N. Kishore and T. Dixit (Varanasi).*

A characterization of semiprime ideals in near rings : *V. Sambasiva Rao (Nagarjuna Nagar).*

Fuzzy connected spaces and weaker forms of fuzzy continuity : *K. K. Azad (Allahabad).*

Disturbances in a semi-infinite piezoelectric medium excited by a tangential electric field : *H. S. Chakraborti (Kalyani).*

Printed at : Paradise Printers, 240, Industrial Area, Chandigarh Ph. 28847